

Harvard University
Department of Economics

General Examination in Macroeconomic Theory

Spring 2009

PLEASE USE A SEPARATE BLUE BOOK FOR EACH PART AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

For those taking the **GENERAL EXAM** in macroeconomic theory:

1. You have **FOUR** hours.
 2. Answer **ALL QUESTIONS** in Parts I, II, III, IV, and V.
 3. Time allotted for each part:
 - I. 48 minutes
 - II. 48 minutes
 - III. 48 minutes
 - IV. 48 minutes
 - V. 48 minutes
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[48 minute question on asset pricing] This question has many parts. If you get stuck on one part, feel free to skip it and continue with the rest. Please write clearly, as your answers will be judged mostly on the quality of your argument (and not just your final mathematical formulae).

- Consider a “static” economy that exists for only two periods. In period 0, agents trade claims (and do *not* consume anything). In period 1 consumption takes place.
 - The economy has two assets that pay out in period 1. (Note that neither asset has continuation value after period 1, since the economy ends at the end of period 1.) The two assets are:
 - A tree that pays out a sure-thing of c in period 1.
 - And a firm with stochastic dividends. With probability $0 < \mu < 1$ the firm generates 1 unit of dividends in period 1. With probability $1 - \mu$ the firm generates 0 dividends in period 1.
 - Assume that there is a representative agent with \ln (natural log) utility.
 - Assume that the economy is in competitive equilibrium.
 - Assume that consumption (in period 1) is the numeraire. We will express the equilibrium price of the firm at date zero as p units of consumption (to be paid in period 1). In other words, if $p = 0.1$, then one can buy Δ fraction of the firm during the trading period (i.e., during period 0) by promising to give the seller a claim that is worth $\Delta \cdot p$ units of consumption in period 1.
1. Use a perturbation argument to explain why the following Euler Equation characterizes the equilibrium price at date zero, p .

$$(1 - p) \mu u'(c + 1) - p (1 - \mu) u'(c) = 0$$

2. Show that

$$\lim_{c \rightarrow \infty} p = \mu.$$

In other words, show that the expected gross “return” on the firm is

$$\lim_{c \rightarrow \infty} E_0 R \equiv \lim_{c \rightarrow \infty} \frac{E_0(\text{payout})}{p} = 1$$

3. Explain the results in part 2 using economic intuition. Specifically, explain why the equilibrium price of the firm is equal to the expected value of the firm (when c is large). Explain why risk aversion does not play a role here, even though the firm has very volatile dividends. (Hint: the data-generating processes are not log-normal, so you need to be selective in the arguments that you use.)
4. Again consider the same setting, but *now* assume that the tree is also risky. This new assumption will be in force for the rest of the problem. Specifically, assume that there are now two states. In the good state, the tree pays $(1 + \alpha)c$ and the firm pays 1 (assume $\alpha > 0$). In the bad state the tree pays c and the firm pays nothing. As before, the good state occurs with probability μ and the bad state occurs with probability $1 - \mu$.
5. Use a perturbation argument to explain why the following (slightly different) Euler Equation characterizes the equilibrium price, p .

$$(1 - p) \mu u'((1 + \alpha)c + 1) - p (1 - \mu) u'(c) = 0$$

6. Show that

$$\lim_{c \rightarrow \infty} p = \frac{\mu}{1 + \alpha(1 - \mu)} < \mu = \text{Expected dividend}$$

In other words, show that the expected gross “return” on the firm is

$$\lim_{c \rightarrow \infty} E_0 R \equiv \lim_{c \rightarrow \infty} \frac{E_0(\text{payout})}{p} = 1 + \alpha(1 - \mu) > 1.$$

7. Explain the results in part 6 using economic intuition. Specifically, explain why the equilibrium price of the firm is less than the expected value of the firm (whether or not c is large). Why does risk aversion now play a role? Contrast this case with the earlier case (when the tree was not risky).

8. Using the equations in part 6, one can generate a high required return on the firm by setting $\alpha = 0.12$ and $\mu = 1/2$. This calibration has the desirable property that it explains the historical equity premium of 0.06. What is wrong with such a calibration. In other words, why is such a calibration inconsistent with other empirical facts?
9. Why is the equity premium *negative* if we instead made $\alpha < 0$?

1 Problem (Optimal Fiscal Policy in the Neoclassical Growth Model)

Consider the Neoclassical Growth Model. Households maximize utility $\sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$ where c_t denotes consumption, l_t denotes labor and β is the rate of time preferences. The production function is denoted by $F(k_t, l_t)$ where k_t is the capital stock. Capital depreciates at rate δ . Firms are competitive and rent labor and capital in every period. A government has to finance an exogenous stream of expenditures g_t in period t (g_t might not be constant). Lump sum taxes are ruled out, but the government can raise linear taxes τ_t^l tax labor income $w_t l_t$ and linear taxes τ_t^k on capital income net of depreciation $k_t (r_t - \delta)$, where w_t and r_t are respectively the wage in period t and the rental rate of capital in period t . Initial capital is taxed at an exogenous rate: $\bar{\tau}_0^k$. The government can also issue debt. Initial debt b_0 is equal to 0. The interest rate on debt in period t is R_t .

ANSWER CAREFULLY AND RIGOROUSLY. QUALITY AND PRECISION OF ANSWER DETERMINES GRADE.

1. Use an arbitrage argument to show that the interest rate R_t on government debt is equal to $(1 - \tau_t^k) (r_t - \delta)$.
2. Suppose that debt entering the period is equal to b_t . Explain why the agents' budget constraint in period t can be written as

$$c_t + b_{t+1} + k_{t+1} = (b_t + k_t) (1 + (r_t - \delta) (1 - \tau_t^k)) + (1 - \tau_t^l) w_t l_t.$$

Derive the government budget constraint.

3. Explain how the agents' per-period budget constraints can be summarized by a single intertemporal condition

$$\sum_{t=0}^{\infty} \frac{c_t - w_t l_t (1 - \tau_t^l)}{\prod_{s=1}^t (1 + (r_s - \delta) (1 - \tau_s^k))} = (1 + (r_0 - \delta) (1 - \bar{\tau}_0^k)) k_0$$

4. Define rigorously a competitive equilibrium with taxes. Explain precisely what Ricardian equivalence means and why it doesn't hold in this model.
5. Explain carefully why and how the optimal fiscal policy $\{\tau_t^l, \tau_t^k\}_{t \geq 0}$ can be characterized by solving the following planning problem

$$\max \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

s.t.

$$c_t + g_t + k_{t+1} \leq F(k_t, l_t) + (1 - \delta) k_t$$

$$\sum_{t=0}^{\infty} \beta^t [c_t U_c(c_t, l_t) + l_t U_l(c_t, l_t)] = [1 + (F_k(k_0, l_0) - \delta)(1 - \bar{\tau}_0^k)] k_0 U_c(c_0, l_0)$$

6. Prove the Chamley Judd result. If the optimal allocation in the planning problem converges to a steady state, then under the optimal fiscal policy, capital taxes converge to zero in the long run: $\lim_{t \rightarrow \infty} \tau_t^k = 0$. Provide intuition for this result. What can be said the behavior of labor taxes in the long run?
7. For this question only, assume that preferences take the form $U(c, l) = \frac{c^{1-\sigma}-1}{1-\sigma} - v(l)$. Show that in this case, a stronger result holds: under the optimal fiscal policy $\tau_t^k = 0$ for all $t \geq 2$. What happens for $t = 1$?
8. In what sense does the optimal fiscal policy feature tax smoothing?
9. Suppose for this question that we introduced human capital in the model. Discuss how the Chamley Judd results might be affected.
10. Suppose that the government could choose an initial tax rate on capital $\tau_0^k \in [0, \bar{\tau}_0^k]$. What would be the optimal choice of τ_0^k ?
11. Discuss what would happen if in period 1, rather than being committed to a path of taxes, the government were given the option to change fiscal policy.

Barro question (48 minutes)

Consider the standard neoclassical growth model, where the representative household maximizes utility over an infinite horizon with time-preference rate $\rho > 0$ and constant population growth rate $n > 0$. The instantaneous utility function is

$u(c) = c^{1-\theta} / (1-\theta)$, where $\theta > 0$ is the magnitude of the elasticity of marginal utility.

The production function is $\hat{y} = f(\hat{k})$, where \hat{y} is output per unit of effective labor and \hat{k} is capital per unit of effective labor. Effective labor grows at the rate $n+x$, where x is the rate of exogenous technical progress. The economy starts with capital per effective worker $\hat{k}(0)$. Capital depreciates at the rate $\delta > 0$. Suppose that there are two kinds of assets: claims on capital and risk-free, short-term bonds.

a. What is the intertemporal elasticity of substitution (IES) for consumption? Explain.

b. How does the (risk-free) real interest rate on bonds relate to the expected rate of return on capital?

c. What is the real interest rate during the transition to the steady state and in the steady state?

d. What is the growth rate of real per capita GDP during the transition to the steady state and in the steady state?

e. What is the transversality condition? What does this condition imply about the relation between the growth rate of real GDP and the real interest rate?

f. How does the condition in **e.** relate to long-term data on growth rates and real rates of return? That is, does the condition in **e.** seem to hold empirically?

g. Suppose that the production function includes a multiplicative random shock, so that the rate of return on capital is uncertain. Does the transversality condition in **e.** still relate to the risk-free real interest rate? Explain.

Question for spring 2009 macro theory generals

BE SURE TO ANSWER ALL PARTS (AND SUB-PARTS) OF THE QUESTION:

By the end of 2008 the U.S. central bank had lowered short-term interest rates to approximately zero (in December the average effective federal funds rate was 0.16 % and the average rate on three-month Treasury bills was just 0.03 %), and short-term rates have remained at that level since then. Yet nonfinancial economic activity has continued to decline. In the meanwhile, Congress has enacted a package of fiscal stimulus measures, and the central bank has continued to implement new facilities for either purchasing debt instruments in the market or advancing funds to enable banks and even some nonbank institutions to increase their lending.

(A) Under what conditions would a monetary policy of maintaining short-term interest rates at the zero lower bound be insufficient, in the absence of additional policy measures, to restore output to its full-employment level? Is it possible for these conditions to persist indefinitely, or would it merely be a matter of a lengthy delay before output would inevitably return to the full-employment level?

(B) If these conditions obtain, whether permanently or merely for a length of time that policymakers regard as unacceptable, so that maintaining zero short-term interest rates is not going to suffice to restore full-employment output, what other avenue(s) might be available to the central bank for this purpose within the confines of conventional monetary policy (in the sense of the central bank's control over short-term interest rates and/or the quantity of its outstanding liabilities)? What would determine whether such a policy strategy would be feasible? What might render it undesirable even if it were feasible?

(C) If all avenues for using conventional monetary policy (again, in the sense of the central bank's control over short-term interest rates and/or the quantity of its outstanding liabilities) to restore full-employment output were either infeasible or regarded as undesirable, what further actions might the central bank take to this end? Specifically, in what way might the kind of actions that the U.S. central bank has recently taken, in purchasing debt instruments and advancing funds to banks and even nonbanks, increase output? Under what conditions would those actions be effective for this purpose? Under what conditions would they prove ineffective?

Be as specific as you can about the analytical framework underlying your answers, as well as any assumptions on which your answers depend.

Economics 2010d: International Macro
Spring 2009: Kenneth Rogoff

1. Bank runs. Consider a Diamond-Dybvig-type model analogous to problem 3 in problem set 2, but with the one difference that there exists a secondary market for illiquid loans.

In particular, consider a simple closed economy model in which there are three dates, 0, 1, 2 and a large number of consumers, each endowed at time 0 with one unit of a consumption good. Consumption can take place either at date 1 or at date 2. Consumers learn only at the beginning of date 1 whether they are an “impatient” consumer, who gets utility only from date-1 consumption, or a “patient” consumer who is indifferent between period 1 and 2 consumption. $\lambda/(1 + \lambda)$ percent of consumers turn out to be impatient and $1/(1 + \lambda)$ turn out to be patient. Impatient consumers turn out to have utility function:

$$u(c_1) = \log c_1$$

where c_1 is an individual’s consumption in period 1. Patient consumers have utility function

$$u(c_1, c_2) = \log(c_2)$$

where c_2 is consumption at date 2. An agent’s type is not observable.

At time zero, the individuals have a choice between storing their good for later use (with no depreciation) or investing their money in a bank. If they invest at bank, they must decide at time 1 whether to withdraw their money (as permitted by their deposit contract) or leave it in. The bank can either hold the deposits as cash (with no net return, and thus gross return 1), or it can invest the money in an illiquid project with gross return $R > 1$, payable only at date 2. The bank can force illiquid projects to be liquidated a period early (in period 1), but it then gets a payout of only $r < 1$.

a. As you may recall (and do not need to show), if there are no bank runs, the optimal contract between banks and depositors has banks paying $c_1 = 1$ to early withdrawers (people who turn out to be impatient), and $c_2 = R$ to people who are able to wait. Why is a bank potentially vulnerable to a run if a lot of patient types suddenly decide that other patient types are pulling their money out in period 1?

b. Would the bank still be vulnerable to a period-1 run if, instead of having to foreclose projects when it runs out of cash, it could sell the projects in a secondary market for price $r' > 1$? What does your answer say about the need for government deposit insurance in an economy with very well functioning secondary markets for private debt?

c. Briefly (you do not need to use any algebra) Can deposit insurance be self-financing in this model? If so, what advantage does the government need to have over the private sector?

2. Debt Buybacks. A small country has foreign debt D coming due at the end of the (one and only) period. Its output Y is uncertain according to the following simple probability distribution: $Y = z$ with probability $1/2$, and $Y = Z$ with probability $1/2$, $\eta z < D < \eta Z$, where η is the percent of the country’s output that can be seized by creditors in the event of default.

a. What is $V(D) = E[\min(\eta Y, D)]$, the market value of the country’s debt (where E denotes expectations)?

b. Will the country’s debt sell at discount? That is, is price $p = V(D)/D < 1$? If so, at what price will it sell?

c. Suppose that in addition to Y , the country also has a small amount of liquid assets (“cash”) C , and that it uses the cash to buy back a portion of its debt at market prices, prior to the realization of Y . Denote the post-buyback debt level as $D - X$, where X is the amount of debt purchased in the buyback. Explain why, if creditor participation in the buyback is totally voluntary, we must have

$$\frac{V(D - X)}{D - X} = \frac{C}{X}$$

where $V(D - X)/(D - X)$ is the post-buyback price. {Hint: Note that C/X is the average amount creditors who sell get.}

d. Why, in general, is the buyback a bad deal for the country here, assuming that creditors have no ability whatsoever to seize the country’s cash assets in the event of default?

3. Please give SHORT answers to ONLY TWO of the following three questions.

- a. Why is the complete markets assumption helpful in motivating the representative agent paradigm so commonly used in macroeconomics and international finance? Assuming zero transactions costs, is complete markets, in general, a sufficient condition for aggregation? How might costs of shipping goods across countries explain the apparent lack of diversification of international portfolios?
- b. What is the evidence that currency unions increase trade? Can you list two or three important critiques of the evidence? Does having a fixed exchange rate appear to produce similar effects on trade quantitatively?
- c. Some classes of models of speculative exchange rate attacks suggest that they might arise in situations of multiple equilibria. What is the potential advantage of such models in explaining the fact that international interest differentials rarely show high expectations of an attack until, at most, weeks before it occurs? How might introducing private information help resolve the multiple equilibria problem? Does such a model potentially explain why a relatively modest degree of capital controls might help prevent speculative attacks?