# Question 5

May 1, 2007

a. (4 points) Capitalists have to solve the following maximization problem:

$$\text{Max} \sum_{t=1}^{\infty} \beta^{t+1} u(c_t),$$

where $c_t = (1-\tau) r k_{t-1} + k_{t-1} - k_t$ ($k_t$ is the capital owned by a capitalist at time $t$).

The first order condition for this maximization problem is:

$$(1-\tau) r u'(c_t) = \beta u'(c_{t+1})$$

that is,

$$\frac{u'(c_t)}{u'(c_{t+1})} = \frac{\beta}{(1-\tau) r}$$

In the steady state, the right-hand side of the equation will be equal to 1. Therefore we have that $r = \frac{\beta}{1-\tau}$.

We also know that capital will be paid its marginal product, hence

$$r = MPL = \alpha \Big(\frac{L}{K}\Big)^{1-\alpha},$$

where $K$ is total capital.

We have $N$ workers who inelastically supply one unit labor; therefore $L = N$.

Solving for K we get

$$K = N \Big(\frac{\alpha(1-\tau)}{\beta}\Big)^{\frac{1}{1-\alpha}}.$$

b. (1 point) The wage is equal to the marginal product of labor:

$$w = MPL = (1-\alpha) \Big(\frac{K}{L}\Big)^{\alpha} = (1-\alpha) \Big(\frac{\alpha(1-\tau)}{\beta}\Big)^{\frac{\alpha}{1-\alpha}}.$$

c. (1 point) The total amount raised is

$$T = \tau r K = \tau \frac{\beta}{1-\tau} N \Big(\frac{\alpha(1-\tau)}{\beta}\Big)^{\frac{1}{1-\alpha}}.$$

Maximizing this amount with respect to the tax rate $\tau$ gives us the following first order condition:

$$(1-\tau)^{\frac{\alpha}{1-\alpha}}\Big(1-\frac{\alpha\tau}{(1-\alpha)(1-\tau)}\Big)=0.$$

The tax rate that generates the highest revenue is therefore $\tau = 1-\alpha$.

d. (2 points) A tax rate equal to 100% would drive capital - and hence tax revenues - to 0.

e. (2 points) Wages plus transfers are equal to

$$\begin{aligned}wN+T=&(1-\alpha)N\Big(\frac{\alpha(1-\tau)}{\beta}\Big)^{\frac{\alpha}{1-\alpha}}+\tau\frac{\beta}{1-\tau}N\Big(\frac{\alpha(1-\tau)}{\beta}\Big)^{\frac{1}{1-\alpha}}\\=&N\Big(\frac{\alpha}{\beta}\Big)^{\frac{\alpha}{1-\alpha}}(1-\tau)^{\frac{\alpha}{1-\alpha}}(1-\alpha+\alpha\tau)\\=&N\Big(\frac{\alpha}{\beta}\Big)^{\frac{\alpha}{1-\alpha}}(1-\tau)^{\frac{\alpha}{1-\alpha}}(1-\alpha+\alpha\tau).\end{aligned}$$

The total amount is decreasing in $\tau$. Let $F(\tau)=N\Big(\frac{\alpha}{\beta}\Big)^{\frac{\alpha}{1-\alpha}}(1-\tau)^{\frac{\alpha}{1-\alpha}}(1-\alpha+\alpha\tau)$. We have $F'(\tau)=\alpha(1-\tau)^{\frac{\alpha}{1-\alpha}}\Big(1-\frac{1-\alpha+\alpha\tau}{(1-\alpha)(1-\tau)}\Big)<0$ for any $\tau\in(0,1)$. Workers are actually better off without taxation ($\tau=0$).