A Two-Country Sticky-Wage Model

Country size and market structure

The world economy consists of two equally-sized countries, Home and Foreign. Firms produce differentiated goods out of differentiated labor inputs indexed by $[0,1]$. Home produces differentiated tradable goods on the interval $[0,1]$, while Foreign's tradables are indexed by $(1,2]$. In addition, each country produces an array of differentiated nontraded goods indexed by $[0,1]$.

Firms

Let $Y(i)$ denote output of differentiated good i and $L(i,j)$ the demand for labor input j by producer i. Home traded goods production is given by

$$Y_{\mathrm{H}}(i) = \left[\int_0^1 L_{\mathrm{H}}(i,j)^{\frac{\phi-1}{\phi}}\mathrm{d}j\right]^{\frac{\phi}{\phi-1}}$$

Home nontraded goods production is identical with $Y_{\mathrm{N}}(i)$ and $L_{\mathrm{N}}(i,j)$ replacing $Y_{\mathrm{H}}(i)$ and $L_{\mathrm{H}}(i,j)$ in the above expression. The Foreign production functions are also identical (

Firm i's demand for labor of type j is

$$L(i,j) = \left[\frac{W(j)}{W}\right]^{-\phi} Y(i),$$

where $W(j)$ is the nominal wage of worker j and W is the exact production-based index of wages, $W = \left[\int_0^1 W(j)^{1-\phi}\mathrm{d}j\right]^{\frac{1}{1-\phi}}$.

Individual preferences

A Home individual of type i maximizes the expected value of

$$U^i = \frac{(C^i)^{1-\rho}}{1-\rho} + \chi \log \frac{M^i}{P} - \frac{K}{\nu}(L^i)^{\nu},$$

where $\rho > 0, \nu > 1$ and

$$L^i \equiv \int_0^1 [L_{\mathrm{H}}(i,j) + L_{\mathrm{N}}(i,j)] \mathrm{d}j$$

K is a random shift in the marginal disutility of work effort that can be interpreted as a (negative) country-wide Home productivity shock. K^*, is distributed symmetrically, though not necessarily independently. The aggregate money supplies, M and M^*, are the other exogenous random variables that impinge on the economy.

In Obstfeld and Rogoff (2000) we focused on the case of log consumption preferences $(\rho = 1)$, but here we generalize to an arbitrary positive ρ.

$$C = C_T^{\gamma} C_N^{1-\gamma} / \gamma^{\gamma} (1-\gamma)^{1-\gamma},$$

$$C_T = 2 C_H^{1/2} C_F^{1/2}.$$

Overall price index

$$P = P_T^{\gamma} P_N^{1-\gamma},$$

Price index for tradable consumption C_T is

$$P_T = P_H^{\frac{1}{2}} P_F^{\frac{1}{2}}$$

consumer demands for individual goods, which depend on relative price with a constant elasticity θ; e.g., Home demand for a typical Home tradable h is

$$C_T(h) = \left[\frac{P_T(h)}{P_H} \right]^{-\theta} C_H.$$

Given the assumed unit elasticity of substitution between Home and Foreign goods, and between traded and nontraded goods, we have: $C_{\mathrm{H}} = \frac{1}{2}(P_{\mathrm{H}}/P_{\mathrm{T}})^{-1}C_{\mathrm{T}}$, $C_{\mathrm{F}} = \frac{1}{2}(P_{\mathrm{F}}/P_{\mathrm{T}})^{-1}C_{\mathrm{T}}$, $C_{\mathrm{T}} = \gamma(P_{\mathrm{T}}/P)^{-1}C$, and $C_{\mathrm{N}} = (1-\gamma)(P_{\mathrm{N}}/P)^{-1}C$. The first-order condition governing money demand is:

$$\frac{M^i}{P} = \chi(C^i)^{\rho}.$$

Asset markets and budget constraints

The only internationally traded asset is a real bond indexed to the composite traded good C_{t}. Thus, domestic firms are entirely domestically owned. We choose not to resort to the popular contrivance of assuming complete Arrow-Debreu contracts internationally since, in general, the presence of such contracts would significantly complicate interpretation of our later strategic results. Also, the marriage of complete contracts and sticky prices into the same model is an uneasy one. In any event, when $\rho = 1$, our economy will turn out to mimic one with complete asset markets.

Given our assumed asset market structure, a Home individual faces the intertemporal budget constraint

$$M^i + PC^i$$
$$= M_0^i + PT + W(i)L^i + \int_0^1 [\Pi_{\mathrm{H}}(j) + \Pi_{\mathrm{N}}(j)]\mathrm{d}j.$$

Here, PT denotes per capita nominal transfers from the Home government, while Π_{H} and Π_{N} denote dividend (profit) payments by firms.

Finally, the government issues money in lump-sum transfers, so that $PT = M - M_0$.

Equilibrium Price and Wage Setting

We assume that workers set nominal wages a period in advance and, ex post, supply the amount of labor that firms demand at the posted nominal wage. The fact that wages are set optimally in response to the government's choice of monetary rule will turn out to be quite important for our later discussion of policy coordination, so we begin by exploring the wage decision.

Optimal wage setting

Use the individual's budget constraint (to eliminate C^i in expected utility $\mathrm{E}U^i$, then substitute for labor supply using labor demand eq.. One derives the first-order condition for the optimal preset nominal wage $W(i)$:

$$W(i) = \left(\frac{\phi}{\phi - 1}\right) \frac{\mathrm{E}\{K(L^i)^{\nu}\}}{\mathrm{E}\left\{\frac{L^i}{P}(C^i)^{-\rho}\right\}}.$$

Absent uncertainty, this would simply give the marginal utility of the real wage as a fixed markup $\phi/(\phi - 1)$ over the marginal disutility of labor.

Price setting*, *the real exchange rate*, *and the terms of trade

As we have already emphasized, monopolistic firms can freely change their products’ prices. However, with constant and identical elasticities of demand at home and abroad, price markups over cost turn out to be the same in both countries, so that, for example,

$$P_{\mathrm{H}} = \left(\frac{\theta}{\theta-1}\right)W = \mathsf{E}P_{\mathrm{H}}^{*}.$$

The relative price of imports moves with the exchange rate, however, so both the real exchange and the terms of trade can still vary. Observe that

$$\text{real exchange rate} \equiv \frac{\mathsf{E}P^{*}}{P} = \frac{\mathsf{E}P_{\mathrm{T}}^{*\gamma}P_{\mathrm{N}}^{*(1-\gamma)}}{P_{\mathrm{T}}^{\gamma}P_{\mathrm{N}}^{(1-\gamma)}} = \left(\frac{\mathsf{E}W^{*}}{W}\right.$$

$$\text{terms of trade} \equiv \frac{\mathsf{E}P_{\mathrm{F}}^{*}}{P_{\mathrm{F}}} = \frac{\mathsf{E}W^{*}}{W}$$

Output market clearing

Because of unit demand elasticities and the budget constraints $P_{\mathrm{T}}C_{\mathrm{T}} = P_{\mathrm{H}}Y_{\mathrm{H}}$ and $\mathcal{E}P_{\mathrm{T}}^{*}C_{\mathrm{T}}^{*} = P_{\mathrm{T}}C_{\mathrm{T}}^{*} = P_{\mathrm{F}}Y_{\mathrm{F}}$, one can easily show that, in all states of nature,

$$C_{\mathrm{T}} = C_{\mathrm{T}}^{*}.$$

If we measure Home spending *in units of tradables* as

$$Z \equiv C_{\mathrm{T}} + \left(\frac{P_{\mathrm{N}}}{P_{\mathrm{T}}}\right)C_{\mathrm{N}},$$

then, because $P_{\mathrm{N}}/P_{\mathrm{T}} = (1-\gamma)C_{\mathrm{T}}/\gamma C_{\mathrm{N}}$,

$$Z = C_{\mathrm{T}}/\gamma = C_{\mathrm{T}}^{*}/\gamma = Z^{*}.$$

Equality of the tradables-denominated spending levels Z and Z^{*} will be helpful in solving the model.

For the subsequent analysis, it is important to observe that for the log consumption case ($\rho = 1$), utility is separable in tradables and nontradables. Thus, when $C_{\mathrm{T}} = C^{*}_{\mathrm{T}}$ ex post, we have perfect international sharing of consumption risks in tradable goods. When $\rho \neq 1$, however, eq. (ref: utility) implies that the marginal utility of *tradables* consumption depends on consumption of nontradables. Thus, $C_{\mathrm{T}} = C^{*}_{\mathrm{T}}$ no longer guarantees internationally equality of the marginal utility of tradables, as efficient risk sharing would require.

Equilibrium preset wages and market equilibrium

In preparation for solving the model, we now substitute the market equilibrium output and pricing conditions into the wage equations Recall the budget constraint $PC = P_{\mathrm{H}}Y_{\mathrm{H}} + P_{\mathrm{N}}Y_{\mathrm{N}} = P_{\mathrm{T}}Z$ and the price markup equations. Also, note that, due to symmetry, in the aggregate, $L = Y_{\mathrm{H}} + Y_{\mathrm{N}}$. We thus obtain

$$\left(\frac{W}{W^*}\right)^{\frac{\nu-(1-\rho)(1-\gamma)}{2}}$$

$$= \frac{\phi\theta}{(\phi-1)(\theta-1)} \frac{\mathrm{E}\{K \mathrm{E}^{\nu/2} Z^{\nu}\}}{\mathrm{E}\left\{\mathrm{E}^{\frac{(1-\rho)(1-\gamma)}{2}} Z^{1-\rho}\right\}}$$

Combining above eq. with its Foreign analog yields:

$$\left(\frac{W}{W^*}\right)^{\nu-(1-\gamma)(1-\rho)}$$

$$= \frac{\mathrm{E}\{K \mathrm{E}^{\nu/2} Z^{\nu}\} \mathrm{E}\left\{\mathrm{E}^{-\frac{(1-\rho)(1-\gamma)}{2}} Z^{1-\rho}\right\}}{\mathrm{E}\{K^* \mathrm{E}^{-\nu/2} Z^{\nu}\} \mathrm{E}\left\{\mathrm{E}^{\frac{(1-\rho)(1-\gamma)}{2}} Z^{1-\rho}\right\}}$$

The preceeding two equations govern the simultaneous determination of wages, expected expenditure, and the expected exchange rate.

A Closed-Form Solution

We now solve the model by assuming that the exogenous shocks $\{m, m^*, \kappa, \kappa^*\}$ are jointly normally distributed, where lower case letters denote (natural) logs so that, e.g., $m \equiv \log M$. We first express the wage setting equations in terms of logs and covariances of logs of the endogenous variables. Later we will write all covariances of endogenous variables in terms of the covariance matrix of exogenous productivity shocks.

Expected relative wages and global spending: *Quasi reduced-form solutions*

A central implication of our modeling approach is that uncertainty has an impact on the expected levels of consumption, output, and the terms of trade through its effect on ex ante wage setting. We first derive these relations.

Representation of productivity disturbances in terms of “difference” and “world” shocks

Though it is not critical to our central message here, it greatly simplifies matters to assume that the Home and Foreign log productivity shocks have *identical* means and variances, so that $\mathrm{E}\kappa = \mathrm{E}\kappa^*$ and $\sigma_\kappa^2 = \sigma_{\kappa^*}^2$. We can then conveniently define the “world” and “difference” productivity shocks as:

$$\kappa_w \equiv \frac{\kappa + \kappa^*}{2}, \;\; \kappa_d \equiv \frac{\kappa - \kappa^*}{2}.$$

Note that because κ and κ^* have identical variances, $\mathrm{Cov}(\kappa_w, \kappa_d) = 0$ and $\sigma_\kappa^2 = \sigma_{\kappa_w}^2 + \sigma_{\kappa_d}^2$. Given the linear-quadratic nature of our setup, the orthogonality of the redefined shocks will later facilitate the separate study of policy rules governing the responses to global and idiosyncratic shocks.

Solutions for mean world spending and terms of trade

Taking logs of wage ratio equation, we obtain

$$\mathrm{E}\tau \equiv \mathrm{E}e + w^* - w$$

$$= \frac{-1}{\nu-(1-\gamma)(1-\rho)}\Big\{[\nu^2 - (1-\gamma)(1-\rho)^2]\sigma_{ze}$$

$$+ \nu\sigma_{\kappa_w e} + 2\nu\sigma_{\kappa_d z},$$

where τ denotes the (log) terms of trade $\mathrm{E}P_{\mathrm{f}}^*/P_{\mathrm{h}}$—making the log real exchange rate $(1-\gamma)\tau$. We solve for the expected log of consumption spending measured in tradables,

$$\mathrm{E}z = \frac{1}{\nu-(1-\rho)}\left\{\begin{array}{c} \omega + \lambda - \frac{\nu}{2[\nu-(1-\rho)]}\sigma_\kappa^2 \\ -\frac{1}{2}[\nu^2 - (1-\rho)^2]\sigma_z^2 \end{array}\right.$$

$$\left.\begin{array}{c} -\frac{1}{8}[\nu^2 - (1-\gamma)^2(1-\rho)^2]\sigma_e^2 \\ -\nu\sigma_{\kappa_w z} - \frac{\nu}{2}\sigma_{\kappa_d e} \end{array}\right\},$$

where ω and λ are constants that depend on the moments of κ and κ^*.

Discussion of the expected spending and terms of trade solutions

Key point is that uncertainty affects the level of output and wages, not just variance.

Ex post spending and the ex post exchange rate

As a final step in expressing the variances of the endogenous variables in terms of the exogenous variables, we solve the sticky wage model for

$$z = \frac{1}{2\rho}(m + m^*) - \frac{1}{2\rho}(w + w^*) - \frac{\log \chi}{\rho} - \frac{1}{\rho}\log\left(\frac{\theta}{\theta - 1}\right),$$

$$e = \frac{m - m^*}{\rho(1 - \gamma) + \gamma} - \frac{(1 - \rho)(1 - \gamma)(w - w^*)}{\rho(1 - \gamma) + \gamma}$$

Once we specify monetary rules for m and m^*, we will be able to present an exact reduced-form solution to the model.

Solving explicitly for expected utility

In studying policy rules, we will look at their welfare implications in the limiting case as $\chi \to 0$ in utility equation. The justification is that expenditure on money services is small relative to that on other goods. [When $\rho = 1$, the solution we present below is exact, even for positive χ. Observe that es that, in equilibrium, $\chi C = M/P$. So in evaluating welfare, we can simply replace the term $\log C + \chi \log \frac{M}{P}$ by $(1 + \chi) \log C$.]

Expected utility when $\rho = 1$

When $\rho = 1$, the utility derived from consumption is simply $\log(C)$ and a Home resident's expected utility (as $\chi \to 0$) takes the form

$$\mathrm{E}U = \mathrm{E}z + \left(\frac{1-\gamma}{2}\right)\mathrm{E}\tau - \frac{(\phi - 1)(\theta - 1)}{\nu\phi\theta}$$

Foreign expected utility is given by

$$\mathrm{E}U^* = \mathrm{E}U - (1-\gamma)\mathrm{E}\tau.$$

The above equations show that while expected consumption measured in tradables, $\mathrm{E}z$, is a common component of Home and Foreign utility, the real exchange rate (proportional to the terms of trade) is a potential source of conflict. Utilities can be expressed exclusively in terms of $\mathrm{E}z$ and $\mathrm{E}\tau$ only in the log case; additional factors enter when $\rho \neq 1$, as we now show.

Expected utility when $\rho \neq 1$

As $\chi \to 0$ in the case $\rho \neq 1$, we evaluate expected utility by calculating

$$\mathrm{E}\left\{ \frac{C^{1-\rho}}{1-\rho} - \frac{K}{\nu} L^{\nu} \right\}$$

$$= \left[\frac{\nu\phi\theta - (1-\rho)(\phi-1)(\theta-1)}{\nu\phi\theta(1-\rho)} \right]$$

$$\mathrm{E}\left\{ Z^{1-\rho} \left(\frac{\mathrm{E}W^*}{W} \right)^{\frac{(1-\rho)(1-\gamma)}{2}} \right\},$$

where

$$\mathrm{E}\left\{ Z^{1-\rho} \left(\frac{\mathrm{E}W^*}{W} \right)^{\frac{(1-\rho)(1-\gamma)}{2}} \right\}$$

$$= \frac{\mathrm{E}\left\{ \exp\left[(1-\rho)z + \frac{(1-\rho)(1-\gamma)}{2} \tau \right] \right\}}{1-\rho}$$

$$= \frac{1}{1-\rho}\exp \begin{array}{c} (1-\rho)\mathsf{E}z + \\ \frac{(1-\rho)(1-\gamma)}{2}\mathsf{E}\tau + \frac{(1-\rho)^2}{2}\sigma_z^2 \end{array}$$

$$+ \frac{(1-\rho)^2(1-\gamma)^2}{8}\sigma_e^2$$

$$+ \frac{(1-\gamma)(1-\rho)^2}{2}\sigma_{ze}.$$

Policy Coordination:

Globally Efficient Precommitment to Monetary Rules

In sticky wage or price models with additional distortions, it need not be the case in general that optimal monetary policy aims simply to mimic the flexible-wage equilibrium. The reason is that the multiple distortions (in the present setup, including wage stickiness, monopoly, and the possible failure of international consumption risk sharing) can interact. Nevertheless, it turns out that for the particular stylized structure we have assumed, the optimal solution to the global cooperation problem indeed replicates the flexible-wage solution in a number of important cases, as we shall now demonstrate.

Optimal cooperation and the flex-wage allocation

If policymakers could cooperate in choosing their domestic monetary policy rules, then with equal weights on national welfares, they would maximize

$$\mathrm{E}V = \tfrac{1}{2}\mathrm{E}U^* + \tfrac{1}{2}\mathrm{E}U.$$

To accomplish this, they would maximize over the coefficients in monetary policy feedback rules of the form

$$\hat{m} = -\delta_d\hat{\kappa}_d - \delta_w\hat{\kappa}_w,$$

$$\hat{m}^* = \delta_d^*\hat{\kappa}_d - \delta_w^*\hat{\kappa}_w,$$

where carets over variables denote innovations, e.g., $\hat{m} \equiv m - \mathrm{E}m$. (Given the loglinear structure of the model, it is plausible to guess that optimal monetary rules will be loglinear too. Of course, $\mathrm{E}\kappa_w = \mathrm{E}\kappa = \mathrm{E}\kappa^*$, so $\mathrm{E}\kappa_d = 0$.)

Expected utilities under flexible and sticky wages

As a first step in understanding cooperation and conflict in the choice of domestic policy rules, we calculate the flexible- and sticky-wage levels of utility in Home and Foreign.

Under flexible wages monetary policy is irrelevant and the level of expected utility, denoted by a tilde, is

$$\mathrm{E}\bullet \;=\; \frac{1}{\nu}\left\{ \begin{array}{c} \log\left[\frac{(\phi-1)(\theta-1)}{\phi\theta} \right] \\ -\frac{(\phi-1)(\theta-1)}{\phi\theta} - \mathrm{E}\kappa \end{array} \right\} = \mathrm{E}\bullet\;{}^{*}$$

when $\rho = 1$, where we have imposed $\mathrm{E}\kappa = \mathrm{E}\kappa^{*}$ and $\sigma_{\kappa}^{2} = \sigma_{\kappa^{*}}^{2}$. For $\rho \neq 1$,

$$\mathrm{E}\bullet \;=\; \mathrm{E}\bullet\;{}^{*} = \left[\frac{\nu\phi\theta - (1-\rho)(\phi-1)(\theta-1)}{\nu\phi\theta(1-\rho)} \right]\exp\left[\frac{(}{\nu\;\cdot} \right.$$

where the constant ω is defined in footnote 4 of th epaper.

Expected Home utility under sticky wages can be written in terms of the flex-wage expected utility levels given above and the economic uncertainties caused by wage rigidity. For $\rho \neq 1$

$$\mathrm{E}U = \left(\mathrm{E}\bullet\right)\exp[(1-\rho)\Omega(\rho)]$$

where $\Omega(\rho)$ is defined (for any $\rho > 0$) as the sum of two terms,

$$\Omega(\rho) = \Omega_w(\rho) + \Omega_d(\rho),$$

such that

$$\Omega_w(\rho) = -\frac{\nu}{2[\nu-(1-\rho)]^2}(\sigma^2_{\kappa_w} + \sigma^2_{\kappa_d}) + \frac{\lambda}{\nu-(1-\rho)} - \frac{\nu}{2}\sigma^2_z - \frac{[\nu-(1-\gamma)^2(1-\rho)]\frac{\nu}{8}\sigma^2_e + \nu\sigma_{\kappa_w z} + \frac{\nu}{2}\sigma_{\kappa_d e}}{\nu-(1-\rho)}$$

and

$$\Omega_d(\rho) = -\frac{(1-\gamma)}{2}\left\{\frac{\nu[\nu-(1-\rho)]\sigma_{ze} + \nu\sigma_{\kappa_w e} + 2\nu\sigma_{\kappa_d}}{\nu-(1-\gamma)(1-\rho)}\right.$$

For $\rho = 1$,

$$\mathrm{E}U = \mathrm{E}\bullet \; + \Omega(1).$$

For Foreign,

$$\mathrm{E}U^{*} = \left(\mathrm{E}\bullet \;\right) \exp[(1-\rho)\Omega^{*}(\rho)]$$

when $\rho \neq 1$, and when $\rho = 1$,
$\mathrm{E}U = \mathrm{E}\bullet \; + \Omega^{*}(1)$, where

$$\Omega^{*}(\rho) = \Omega_{w}(\rho) - \Omega_{d}(\rho).$$

Obviously, $\Omega_{w}(\rho)$ is a symmetric component of world utility that affects Home and Foreign welfare equally. E.g., a rise in the variance of world spending (σ_{z}^{2}) or the exchange rate (σ_{e}^{2}) has symmetrical negative expected utility effects upon Home and Foreign.

The term $\Omega_{d}(\rho)$ is an asymmetric utility component that affects Home and Foreign in opposite ways. For example, a rise in $\sigma_{\kappa_{w}e}$ hurts Home because it becomes more likely that demand for Home output will be unexpectedly high when there is an unexpectedly high global aversion to effort. But that same change represents a commensurate benefit to Foreign.

Multiple distortions and the efficiency of the flexible-wage equilibrium

Is it efficient (from an ex ate standpoint) to have monetary policy rules aim to mimic the flexible-wage equilibrium, as in 1980s style of rational-expectations monetary models? In general, the answer is not trivial, as we have noted, since wage stickiness is not the only distortion here. In this subsection we establish a sufficient condition under which optimal cooperative choice of the policy rules results in the flexible-wage allocation ex post.

Proposition *If the flexible-wage allocation is constrained Pareto efficient (subject to the constraint that labor supplies are at monopolistic levels), a global monetary policy rule that gives the same real allocation as under flexible wages is efficient.*

Under the assumptions of Proposition 1, targeting the flexible-wage allocation is also the *optimal* cooperative policy given the assumed 50-50 weights on country utility in the planner objective function The reason is that E• $=$ E• *; Later, however, we will see that Proposition 1 gives a sufficient condition for optimality *regardless* of the weights.

Optimal cooperation

The proposition just proved allows a quick but partial characterization of optimal policies. When all productivity shocks are world shocks (that is, $\hat{\kappa}_d \equiv 0$), or when $\rho = 1$, the sharing of tradable consumption risks is efficient and there is no global distortion to the flexible-wage equilibrium other than the ones caused by monopoly (which enter separably). In these latter cases, therefore, we would expect optimal cooperative policies to target the flex-wage allocation. More generally, however, international risk sharing may not be efficient and economic distortions (other than our particular specification of monopoly power) may interact with the sticky-wage distortion to dictate ex post deviations from the flexible-wage allocation.

To better understand the general case, we now solve explicitly for the efficient policy rules.

Noncooperative Choice of Policy Rules

In designing their monetary rules and institutions, countries seldom ask what impact domestic institutional changes will have on welfare abroad.

We show in this section that when the optimal cooperative policy rules target the flexible-wage equilibrium ex post, those rules are also Nash equilibrium rules.

A corollary of this result is that countries' responses to global, internationally symmetric, shocks do not raise problems of coordination; only asymmetric shocks may be problematic.

Nash equilibrium in policy rules

The first-order conditions for Home's problem are the same as those for the problem

$$\max_{\delta_d,\delta_w} \underbrace{\mathsf{E}z + \frac{(1-\rho)}{2}\sigma_z^2 + \frac{(1-\rho)(1-\gamma)^2}{8}\sigma_e^2}_{\text{global component}} + \underbrace{\frac{(1-\gamma)}{2}\mathsf{E}\tau + \frac{(1-\gamma)}{2}}_{\text{country-specific comp}}$$

given δ_d^* and δ_w^*. Foreign's effective objective function is simply the global component above *less* the country-specific component. IWe can pose Home's problem equivalently as

$$\max_{\delta_d,\delta_w} \underbrace{\frac{\omega}{\nu-(1-\rho)} + \Omega_w(\rho)}_{\text{global component}} + \underbrace{\Omega_d(\rho)}_{\text{country-specific component}}$$

Starting at the cooperative equilibrium, a small move of δ_d^{*coop} away from δ_d^{coop}, say, has no first-order impact on the global component of Home expected utility because that term is maximized by a global planner; r. Yet, given the Foreign policy rule, Home might still wish to change its own rule, reaping a net *domestic* gain by shifting the utility-relevant terms Eτ and σ_{ze} in its favor while lowering the global component of welfare by less. In that case, of course, Foreign would lose more than Home gains, and, starting at the cooperative equilibrium, Foreign would face a symmetrical incentive to make a "beggar-thy-neighbor" change in its policy rule.

The Nash equilibrium, like the cooperative one, is symmetric, so $\delta_d^{Nash} = \delta_d^{*Nash}$ and $\delta_w^{Nash} = \delta_w^{*Nash}$. Going beyond this observation, our next result tells us that there is no individual incentive for countries to defect from the cooperative equilibrium when that equilibrium mimics the flexible-wage equilibrium ex post.

Proposition *In the Nash monetary policy rule setting equilibrium,* $\delta_d^{Nash} = \delta_d^{coop} = \delta_d^{flex}$ *when* $\rho = 1$, *and* $\delta_w^{Nash} = \delta_w^{coop} = \delta_w^{flex}$ *for any* $\rho > 0$.

Proposition 2 shows that when the flexible-wage equilibrium is constrained-efficient with respect to the monopoly distortions, Home doesn't gain by unilaterally moving its policy rule away from cooperation. Constrained efficiency always holds when $\rho = 1$, and it holds for any $\rho > 0$ when all shocks are symmetric. Thus the cooperative equilibrium—when it mimics the flexible-wage equilibrium—is also the Nash equilibrium of the rule-setting game. But notice that our result actually is stronger than this. In fact, the proposition states that Home never gains from changing its response to symmetric shocks even when asymmetric shocks can occur and $\rho \neq 1$. This "separability" property follows from the basic linear-quadratic nature of our model, coupled with the orthogonality of the "world" and "difference" shocks.

Regarding the Nash response to idiosyncratic shocks, we have

Proposition *In the Nash monetary policy rule setting equilibrium,* $\delta_d^{flex} > \delta_d^{Nash} > \delta_d^{coop}$ *when* $\rho < 1$ *and* $\delta_d^{flex} < \delta_d^{Nash} < \delta_d^{coop}$ *when* $\rho > 1$.

The proof of Proposition 2 in appendix 1 confirms our earlier claim that under the assumptions of Proposition 1, it is optimal for a global planner to target the flexible-wage equilibrium regardless of the country welfare weights in the objective function (ref: planner). Thus, we have:

Corollary *If the flexible-wage allocation is constrained Pareto efficient, a global monetary policy that gives the same real allocation as under flexible wages is optimal even for a supranational planner who favors one country over the other.* ■

Can one plausibly generate big coordination gains?

One way to assess the quantitative importance of the gain from coordination when $\rho \neq 1$ and $\sigma^2_{\kappa_d} > 0$ is to simulate our model numerically. To that end, we assume $\sigma^2_{\kappa_d} = \sigma^2_{\kappa_w} = .01,$ that $\gamma = 0.6$, and that $\nu = 1.5$ (the value used by Chari, Kehoe, and McGrattan). For different values of ρ, table 1 calculates three numbers: (i) the gain from monetary policies that target the flexible-wage equilibrium, compared with policies that hold money supplies constant; (ii) the gain from moving from flex-wage policies to the cooperative equilibrium; and (iii) the ratio of (ii) to (i). The gains (i) and (ii) are expressed as *percentages* of output. Note that because the Nash equilibrium policy responses lie between the flex-wage and cooperative responses, the ratio (iii) is strict upper bound on the gains to cooperative versus Nash behavior in rule setting.

Table 1: **Gains from stabilization and coordination**

Measure	$\rho = 0.5$	$\rho = 1$	$\rho = 2$
(i) Stabilization gain	1.28	0.67	0.30
(ii) Coordination gain	4.5×10^{-3}	0	2.9×10
(iii) Ratio (ii)/(i)	3.5×10^{-3}	0	9.7×10

However, the net gain to cooperation versus simply targeting the flex-wage allocation is uniformly tiny. Only at unrealistically high values of ρ, at which the gain to stabilization is very small, does the gain from coordination climb to even a tenth of the gain from stabilization. Thus, the discrepancy between the Nash and cooperative equilibria seems truly negligible in welfare terms, even when measured against a limited scope for stabilization.

Comparisons with Earlier Literature

Oudiz and Sachs (1984),

Jensen (2000), Persson and Tabellini (1995) and Rogoff (1985), Drazen (2000) e

Conclusions

Modern models of monetary policy transmission suggest a number of channels that might lead countries to choose monetary rules that are optimal from a national perspective but not from a global perspective. While in principle this problem perhaps can be addressed with properly designed domestic monetary institutions—as Jensen (2000) and Persson and Tabellini (1995, 2000) have suggested—in practice, spillover effects appear to receive only minimal consideration.

We have shown that, surprisingly, this lack of coordination may not be a first-order problem. As domestic monetary rules improve, and as international asset markets become more complete, the outcome of a Nash monetary rule-setting game begins to approximate the outcome of a cooperative one. In principle, the current system of monetary arrangements may evolve to one that is (nearly) optimal from a stabilization point of view, without any major institutional innovations at the international level.