Obstfeld and Rogoff, ”New Directions for Stochastic Open Economy Models

The expenditure-switching effects of exchange rate changes in traditional Keynesian models

$$TOT = \frac{P_{\mathrm{F}}}{\mathcal{E}P^{*}_{\mathrm{H}}}.$$

The effects of exchange rate changes with local-currency pricing

Reservations about the PTM-LCP approach

Notwithstanding their potential to mimic a selected subset of business-cycle facts, we find recent models built on the PTM-LCP approach highly implausible because their assumptions and predictions appear grossly inconsistent with many other facts.

- A large fraction of measured deviations from the law of one price is the result of nontradable components incorporated in consumer price indexes for supposedly tradable goods.
- The likely time horizon over which trade invoicing induces price stickiness appears too brief to have a large impact on macroeconomic interactions at business-cycle frequencies.
- The direct evidence on currency invoicing is largely inconsistent with the view that exporters set prices predominantly in importers' currencies. The percentages of exports and imports, respectively, denominated in home currency are: Japan (40, 17), Germany (77, 56), France (55, 47), United Kingdom (62, 43), Italy (40, 34), Netherlands (43, 39).[5]
- International evidence on markups also seems consistent with a predominance of invoicing in exporters' home currencies.

Empirics:

Comovements of exchange rates and bilateral relative competitiveness

Table 2 Correlations of monthly log changes in ex
rates and relative export competitiveness, 1982

	Germany	Italy	Japan	U.K.	U
Canada	0.87	0.85	0.74	0.88	0.
Germany		0.65	0.67	0.74	0.
Italy			0.73	0.79	0.
Japan				0.76	0.
U.K.					0.

Source: Based on data from IMF, International Financial Statistics.

A stochastic sticky-wage model with traded and nontraded goods

$$Y_{\mathrm{H}}(i) = \left[\int_0^1 L_{\mathrm{H}}(i,j)^{\frac{\phi-1}{\phi}} \mathrm{d}j \right]^{\frac{\phi}{\phi-1}}$$

$$Y_{\mathrm{N}}(i) = \left[\int_0^1 L_{\mathrm{N}}(i,j)^{\frac{\phi-1}{\phi}} \mathrm{d}j \right]^{\frac{\phi}{\phi-1}}.$$

$$W = \left[\int_0^1 W(i)^{1-\phi} \mathrm{d}i \right]^{\frac{1}{1-\phi}}.$$

$$L(i,j) = \left[\frac{W(i)}{W} \right]^{-\phi} Y(j),$$

Individual preferences ($\nu \geq 1$.)

$$U^i = \log(C^i) + \frac{\chi}{1-\varepsilon} \left(\frac{M^i}{P} \right)^{1-\varepsilon} - \frac{K}{\nu} (L^i)^{\nu},$$

$$L^i \equiv \int_0^1 [L_{\mathrm{H}}(i,j) + L_{\mathrm{N}}(i,j)] \mathrm{d}j$$

$$C = \frac{C_{\mathrm{T}}^{\gamma} C_{\mathrm{N}}^{1-\gamma}}{\gamma^{\gamma} (1-\gamma)^{1-\gamma}},$$

Preferences over Home and Foreign *tradable* products have an Armington form,

$$C_{\mathrm{t}} = 2C_{\mathrm{H}}^{\frac{1}{2}} C_{\mathrm{F}}^{\frac{1}{2}}.$$

$$C_{\mathrm{H}} = \left[\int_0^1 C_{\mathrm{T}}(j)^{\frac{\theta-1}{\theta}} \mathrm{d}j \right]^{\frac{\theta}{\theta-1}},$$

$$C_{\mathrm{F}} = \left[\int_1^2 C_{\mathrm{t}}(j)^{\frac{\theta-1}{\theta}} \mathrm{d}j \right]^{\frac{\theta}{\theta-1}},$$

$$C_{\mathrm{N}} = \left[\int_0^1 C_{\mathrm{N}}(j)^{\frac{\theta-1}{\theta}} \mathrm{d}j \right]^{\frac{\theta}{\theta-1}}.$$

Consumption-based price indices

$$P = P_{\mathrm{T}}^{\gamma} P_{\mathrm{N}}^{1-\gamma}$$

$$P_{\mathrm{T}} = P_{\mathrm{H}}^{\frac{1}{2}} P_{\mathrm{F}}^{\frac{1}{2}}.$$

The model is rigged so that there is constant elasticity of demand for everything:

$$C_{\mathrm{T}}(h) = \left[\frac{P_{\mathrm{T}}(h)}{P_{\mathrm{H}}} \right]^{-\theta} C_{\mathrm{H}}$$

(demand for typical Home tradable produc

$$C_{\mathrm{T}}(f) = \left[\frac{P_{\mathrm{T}}(f)}{P_{\mathrm{F}}} \right]^{-\theta} C_{\mathrm{F}}$$

(demand for typical Foreign tradable produ

$$C_{\mathrm{N}}(h) = \left[\frac{P_{\mathrm{N}}(h)}{P_{\mathrm{N}}} \right]^{-\theta} C_{\mathrm{N}}$$

(demand for typ. Home nontradable *h*),

$$C_{\mathrm{H}} = \tfrac{1}{2}\left(\frac{P_{\mathrm{H}}}{P_{\mathrm{T}}} \right)^{-1} C_{\mathrm{T}}, \quad C_{\mathrm{F}} = \tfrac{1}{2}\left(\frac{P_{\mathrm{F}}}{P_{\mathrm{T}}} \right)^{-1} C_{\mathrm{T}},$$

$$C_{\mathrm{T}} = \gamma\left(\frac{P_{\mathrm{T}}}{P} \right)^{-1} C, \quad C_{\mathrm{N}} = (1-\gamma)\left(\frac{P_{\mathrm{N}}}{P} \right)^{-1} C.$$

$$\frac{1}{C^i} = \chi\left(\frac{M^i}{P} \right)^{-\varepsilon}.$$

Asset markets and budget constraints

(assumes no trade in risky assets, but equilibrium turns out to mimic complete nominal contracts model. This is only true because we have assumed elasticity of substitution between home and foreign goods of one; we discuss more general case later.)

$$M^i + PC^i = M_0^i + PT + W(i)L^i + \int_0^1 [\Pi_{\mathrm{H}}(j) + \Pi_{\mathrm{N}}(j)]\mathrm{d}j,$$

Government's budget constraint

$$PT = M_1 - M_0$$

Optimal wage setting (We assume that nominal wages must be set a period in advance while prices are flexible. The wage setter takes into account risk of shocks to money and productivity.)

The first-order condition for wages

$$W(i) = \left(\frac{\phi}{\phi - 1} \right) \frac{\mathsf{E}\{K(L^i)^\nu\}}{\mathsf{E}\left\{ \frac{L^i}{PC^i} \right\}}.$$

has simple intuitive intrepretation as in the nonstochastic case.

$$W = \left(\frac{\phi}{\phi - 1} \right) \frac{\mathsf{E}\{K(L)^\nu\}}{\mathsf{E}\left\{ \frac{L}{P}(C)^{-\rho} \right\}}.$$

$$W = \left(\frac{\phi}{\phi - 1} \right) \frac{\mathsf{E}\{K\}(\mathsf{E}\{L\})^{\nu-1}}{(\mathsf{E}\{C\})^{-\rho}\mathsf{E}\left\{ \frac{1}{P} \right\}} \exp(\xi).$$

The factor ξ, entirely due to uncertainty, is given by

$$\xi \equiv \frac{\nu(\nu - 1)}{2}\sigma_l^2 - \frac{\rho(\rho + 1)}{2}\sigma_c^2$$

$$+ \nu\sigma_{\kappa l} + \rho\sigma_{cl} - \rho\sigma_{cp} + \sigma_{lp}.$$

Price setting, the real exchange rate, and the terms of trade

$$P_{\mathrm{N}} = P_{\mathrm{H}} = \left(\frac{\theta}{\theta-1}\right)W,$$

$$P^*_{\mathrm{N}} = P^*_{\mathrm{F}} = \left(\frac{\theta}{\theta-1}\right)W^*,$$

$$P^*_{\mathrm{H}} = \frac{1}{\mathcal{E}}\left(\frac{\theta}{\theta-1}\right)W = \frac{1}{\mathcal{E}}P_{\mathrm{H}}, \quad P_{\mathrm{F}} = \mathcal{E}\left(\frac{\theta}{\theta-1}\right)W^* = \mathcal{E}P^*_{\mathrm{f}}$$

$$\text{Real ex. rate} \equiv \frac{\mathcal{E}P^*}{P} = \frac{\mathcal{E}P_{\mathrm{T}}^{*\gamma}P_{\mathrm{N}}^{*(1-\gamma)}}{P_{\mathrm{T}}^{\gamma}P_{\mathrm{N}}^{(1-\gamma)}}$$

$$= \frac{\mathcal{E}\left(P_{\mathrm{H}}^{*\frac{1}{2}}P_{\mathrm{F}}^{*\frac{1}{2}}\right)^{\gamma}P_{\mathrm{N}}^{*(1-\gamma)}}{\left(P_{\mathrm{H}}^{\frac{1}{2}}P_{\mathrm{F}}^{\frac{1}{2}}\right)^{\gamma}P_{\mathrm{N}}^{(1-\gamma)}}$$

$$= \left(\frac{\mathcal{E}W^*}{W}\right)^{1-\gamma},$$

$$TOT \equiv \frac{\mathcal{E}P^*_{\mathrm{F}}}{P_{\mathrm{H}}} = \frac{\mathcal{E}W^*}{W},$$

Market clearing and the current account

$$P_{\mathrm{H}}Y_{\mathrm{H}} = \tfrac{1}{2}P_{\mathrm{T}}C_{\mathrm{T}} + \tfrac{1}{2}\mathcal{E}P_{\mathrm{T}}^{*}C_{\mathrm{T}}^{*},$$

$$P_{\mathrm{F}}Y_{\mathrm{F}} = \tfrac{1}{2}P_{\mathrm{T}}C_{\mathrm{T}} + \tfrac{1}{2}\mathcal{E}P_{\mathrm{T}}^{*}C_{\mathrm{T}}^{*},$$

$$\frac{P_{\mathrm{H}}}{P_{\mathrm{F}}} = \frac{Y_{\mathrm{F}}}{Y_{\mathrm{H}}}$$

$$C_{\mathrm{T}} = C_{\mathrm{T}}^{*}$$

$$Z \equiv C_{\mathrm{T}} + \left(\frac{P_{\mathrm{N}}}{P_{\mathrm{T}}}\right)C_{\mathrm{N}}.$$

$$\frac{P_{\mathrm{N}}}{P_{\mathrm{T}}} = \left(\frac{1-\gamma}{\gamma}\right)\frac{C_{\mathrm{T}}}{C_{\mathrm{N}}},$$

$$Z = C_{\mathrm{T}}/\gamma = C_{\mathrm{T}}^{*}/\gamma = Z^{*}$$

This last condition very special to the model but allows for convenient closed form solution to the model.

Equilibrium preset wages

$$W = \left(\frac{\phi}{\phi - 1}\right)\frac{\mathsf{E}\{K(Y_{\mathrm{H}} + Y_{\mathrm{N}})^{\nu}\}}{\mathsf{E}\left\{\frac{Y_{\mathrm{H}} + Y_{\mathrm{N}}}{PC}\right\}}$$

$$\left(\frac{W}{W^*}\right)^{\nu/2} = \frac{\phi\theta}{(\phi - 1)(\theta - 1)}\mathsf{E}\{K\mathsf{E}^{\nu/2}Z^{\nu}\}.$$

$$\left(\frac{W^*}{W}\right)^{\nu/2} = \frac{\phi\theta}{(\phi - 1)(\theta - 1)}\mathsf{E}\{K^*\mathsf{E}^{-\nu/2}Z^{\nu}\}.$$

$$\left(\frac{W}{W^*}\right)^{\nu} = \frac{\mathsf{E}\{K\mathsf{E}^{\nu/2}Z^{\nu}\}}{\mathsf{E}\{K^*\mathsf{E}^{-\nu/2}Z^{\nu}\}}.$$

The above three equations (two are independent) determine the equilibrium of the system. They are highly nonlinear but have a closed form solution.

A closed-form solution

Expected relative wages and global spending: *Quasi reduced-form solutions*

$$\mathrm{E}e + w^* - w = -\nu\sigma_{ze} - \tfrac{1}{2}(\sigma_{\kappa e} + \sigma_{\kappa^* e})$$

$$+ (\sigma_{\kappa^* z} - \sigma_{\kappa z}) \equiv \mathrm{E}\tau.$$

$$\mathrm{E}z = \omega - \frac{\nu}{2}\sigma_z^2 - \frac{\nu}{8}\sigma_e^2 -$$

$$\frac{1}{2}(\sigma_{\kappa z} + \sigma_{\kappa^* z}) - \frac{1}{4}(\sigma_{\kappa e} - \sigma_{\kappa^* e}),$$

$$\omega \equiv$$

$$\frac{1}{\nu}\left\{\log\left[\frac{(\phi-1)(\theta-1)}{\phi\theta}\right] - \mathrm{E}\kappa - \frac{1}{2}\sigma_\kappa^2\right\}.$$

Ex post spending, *the ex post exchange rate*, *and nominal wage levels*

$$z = \frac{\varepsilon}{2}(m + m^*) - \frac{\varepsilon}{2}(w + w^*)$$

$$- \log\chi - \varepsilon\log\left(\frac{\theta}{\theta - 1}\right).$$

$$e = \frac{\varepsilon(m - m^*)}{1 - \gamma + \gamma\varepsilon} - \frac{(\varepsilon - 1)(1 - \gamma)(w - w^*)}{1 - \gamma + \gamma\varepsilon}.$$

$$w = \mathsf{E}m - \log\left(\frac{\theta}{\theta - 1}\right) - \frac{(\mathsf{E}z + \log\chi)}{\varepsilon} - \frac{(1-\gamma) + \gamma\varepsilon}{\varepsilon}\left(\frac{\mathsf{E}\tau}{2}\right),$$

$$w^* = \mathsf{E}m^* - \log\left(\frac{\theta}{\theta - 1}\right) - \frac{(\mathsf{E}z + \log\chi)}{\varepsilon} + \frac{(1-\gamma) + \gamma\varepsilon}{\varepsilon}\left(\frac{\mathsf{E}\tau}{2}\right).$$

Solutions for variances

results for the case when monetary policy does not respond to productivity shocks so that $\sigma_{\kappa z}, \sigma_{\kappa^* z}, \sigma_{\kappa e},$ and $\sigma_{\kappa^* e}$ are all zero:

$$\sigma_e^2 : \quad \left(\frac{\varepsilon}{1 - \gamma + \gamma\varepsilon}\right)^2 (\sigma_m^2 - 2\sigma_{mm^*} + \sigma_{m^*}^2)$$

$$\sigma_z^2 : \quad \frac{\varepsilon^2}{4}(\sigma_m^2 + 2\sigma_{mm^*} + \sigma_{m^*}^2)$$

$$\sigma_{ze} : \quad \left(\frac{\varepsilon^2}{1 - \gamma + \gamma\varepsilon}\right)\frac{(\sigma_m^2 - \sigma_{m^*}^2)}{2}$$

Solution for utility

This turns out to be remarkably simple if we make use of the FOC for wages to substitute out for leisure in expected utility:

$$\mathsf{E}\{C^{1-\rho}\} = \frac{\phi\theta}{(\phi-1)(\theta-1)}\mathsf{E}\{KL^{\nu}\}.$$

Then we can write

$$\begin{aligned}\mathsf{E}U &= \mathsf{E}\left\{\frac{C^{1-\rho}}{1-\rho} - \frac{K}{\nu}L^{\nu}\right\} \\ &= \mathsf{E}\left\{\frac{C^{1-\rho}}{1-\rho} - \frac{(\phi-1)(\theta-1)}{\nu\phi\theta}C^{1-\rho}\right\} \\ &= \frac{\nu\phi\theta-(\phi-1)(\theta-1)(1-\rho)}{\nu\phi\theta}\mathsf{E}\left\{\frac{C^{1-\rho}}{1-\rho}\right\}.\end{aligned}$$

$$\mathsf{E}U = \mathsf{E}c - \frac{(\phi-1)(\theta-1)}{\nu\phi\theta} =$$

$$\log\mathsf{E}C - \tfrac{1}{2}\sigma_c^2 - \frac{(\phi-1)(\theta-1)}{\nu\phi\theta},$$

$$
\mathsf{E}U = \mathsf{E}z + \left(\frac{1-\gamma}{2}\right)\mathsf{E}\tau - \frac{(\phi-1)(\theta-1)}{\nu\phi\theta}
$$

$$
= \frac{1}{\nu}\left\{\begin{array}{c} \log\left[\dfrac{(\phi-1)(\theta-1)}{\phi\theta}\right] - \\ \dfrac{(\phi-1)(\theta-1)}{\phi\theta} - \mathsf{E}\kappa \end{array}\right\} + \Omega,
$$

$$
\Omega \equiv -\frac{\nu}{2}\sigma_z^2 - \frac{\nu}{8}\sigma_e^2 - \frac{1}{2\nu}\sigma_\kappa^2 - \frac{(1-\gamma)\nu}{2}\sigma_{ze}
$$

$$
- \frac{(2-\gamma)}{2}(\sigma_{\kappa z} + \frac{1}{2}\sigma_{\kappa e}) -
$$

$$
\frac{\gamma}{2}(\sigma_{\kappa^* z} - \frac{1}{2}\sigma_{\kappa^* e})
$$

Efficient monetary policies and exchange rate regimes

We can actually think about optimal exchange rate regimes in this model. First, we ask what the most efficient regime would be.

Efficient monetary policy rules

Flexible-price equilibrium

$$\tilde{L} = \tilde{L}_{\mathrm{H}} + \tilde{L}_{\mathrm{N}} = \left[\frac{(\phi - 1)(\theta - 1)}{K\phi\theta} \right]^{1/\nu},$$

$$\tilde{L}^{*} = \tilde{L}_{\mathrm{F}}^{*} + \tilde{L}_{\mathrm{N}}^{*} = \left[\frac{(\phi - 1)(\theta - 1)}{K^{*}\phi\theta} \right]^{1/\nu},$$

$$\frac{\mathrm{d}l}{\mathrm{d}\kappa} = \frac{\mathrm{d}l^{*}}{\mathrm{d}\kappa^{*}} = -\frac{1}{\nu} < 0.$$

$$\mathrm{E}\bullet = \mathrm{E}\bullet^{*}$$

$$= \frac{1}{\nu}\left\{ \begin{array}{c} \log\left[\frac{(\phi-1)(\theta-1)}{\phi\theta} \right] \\ -\frac{(\phi-1)(\theta-1)}{\phi\theta} - \mathrm{E}\kappa \end{array} \right\},$$

Dual distortions and the efficiency of the flexible-price equilibrium

Proposition *A global monetary policy that gives the same real allocation as under flexible wages is efficient.*

proof

$$m = \mathrm{E}m + \frac{1}{2\nu\varepsilon}\left\{ \begin{array}{c} \gamma(\varepsilon - 1)(\kappa^* - \mathrm{E}\kappa^*) \\ -[2 + \gamma(\varepsilon - 1)](\kappa - \mathrm{E}\kappa) \end{array} \right\},$$

$$m^* = \mathrm{E}m^* + \frac{1}{2\nu\varepsilon}\left\{ \begin{array}{c} \gamma(\varepsilon - 1)(\kappa - \mathrm{E}\kappa) \\ -[2 + \gamma(\varepsilon - 1)](\kappa^* - \mathrm{E}\kappa^*) \end{array} \right\}.$$

$$m + m^* = \mathrm{E}m + \mathrm{E}m^* - \frac{1}{\nu\varepsilon}\left[\begin{array}{c} (\kappa - \mathrm{E}\kappa) \\ +(\kappa^* - \mathrm{E}\kappa^*) \end{array} \right].$$

Welfare under alternative monetary regimes

$$\mathrm{E}U^{Optimal\ float} = \frac{1}{\nu}\left\{ \begin{array}{c} \log\left[\frac{(\phi-1)(\theta-1)}{\phi\theta}\right] \\ -\frac{(\phi-1)(\theta-1)}{\phi\theta} - \mathrm{E}\kappa \end{array} \right\}.$$

$$\mathrm{E}U^{World\ monetarism} = \mathrm{E}U^{Optimal\ float} - \frac{1}{2\nu}\sigma_\kappa^2.$$

$$z = -\alpha(\kappa + \kappa^*).$$

$$-\frac{1}{2\nu}\sigma_\kappa^2 - \frac{\nu}{2}\sigma_z^2 - \sigma_{\kappa z} =$$

$$-\left(\frac{1}{2\nu} + \nu\alpha^2 - \alpha\right)\sigma_\kappa^2,$$

$$\alpha = \frac{1}{2\nu},$$

$$z = -\frac{(\kappa + \kappa^*)}{2\nu}$$

$$\mathrm{E}U^{Optimal\ fix} =$$

$$\mathrm{E}U^{Optimal\ float} - \frac{1}{4\nu}\sigma_\kappa^2 > \mathrm{E}U^{World\ monetarism}.$$