

Harvard University
Department of Economics

General Examination in Microeconomic Theory

Fall 2008

1. You have **FOUR** hours.
2. Answer all questions

PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.
PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

There are two occupations in this economy and one occupation pays higher.

1 – Give 3 different explanations how this wage difference can be an equilibrium phenomenon.

2 – Assume that the productivity of one occupation increases what happens to wages and employment in the two industries

3 – What impact will this change have in productivity in measured GDP? Will this measured changed over-state or under-state welfare changes?

4 – What impact will a proportional income tax rate have on income earning in the two occupations?

"Job Market Signalling" by A. Michael Spence is one of the most famous papers in economics. A game theoretic interpretation of this paper is as follows: There is one worker and two firms $i = 1, 2$. The worker has a productivity of θ_L or θ_H which she knows. The firms know the probabilities of these two productivity values, p and $1 - p$ but that is all they know. The worker can undertake to get education at any level $e \in [0, \bar{e}]$. Education adds nothing to productivity. It is more costly for θ_L people to get any given level of e than it is for θ_H people. Let $c(e, \theta)$ be this cost function.

The "signalling game" is played as follows: First the worker observes the value of θ . Then the worker chooses e . Then the two firms, simultaneously, observe e and each makes a wage offer $w_i(e)$. Then the worker chooses the higher of the two wage offers, or randomizes among them if they are equal. Payoffs are $w_{\max}(e) - c(e, \theta)$ for the worker, where w_{\max} is the higher of the two wage offers, and $\theta - w_{\max}(e)$ for the higher-paying firm and 0 for the lower paying firm. Firms want to maximize the expected value of their payoffs.

1) What are the Bayesian Perfect equilibria of this game? Carefully explain what a strategy is for the worker. Examine only equilibria where both firms play the same strategy and restrict attention to only pure strategy equilibria.

2) What is wrong with some of these equilibria? (Explain in words.) How has game theory tried to reformulate the concept of Perfect Bayesian equilibrium to get rid of them?

3) What are the "most refined" of the equilibria? That is, of the various equilibrium refinements you know, which results in the smallest set of equilibria for this example.

4) In the real world, education takes time. The "education level" e that a worker can select is really a length of time that the worker will stay in school. Suppose that the cost of education is paid as a flow, θ per unit time, for as long a time that the worker stays in school. Suppose further that no worker can make an irrevocable commitment to stay in school. Therefore the game proceeds (taking a discrete time version of this story) in a sequence of steps where the worker enrolls in school for one (short) period, then firms make wage offers based on whether or not the worker has been in school (for that short period), then the worker chooses whether to accept the wage offer and leave school or to reject it and stay in school, anticipating a higher wage offer later, etc. Without writing down a formal model, argue that the "most refined equilibrium" you have written down in part 3) above is **not** an equilibrium of this sequential game involving incremental decisions to stay in school or leave.

- i. Consider an exchange economy. Explain what is meant by Walrasian equilibrium, Pareto optimality and the core for such an economy. Prove that every competitive equilibrium is Pareto optimal and in the core. Illustrate the relationship between these concepts using an Edgeworth box. Show, diagrammatically or otherwise, that the core becomes smaller as the economy grows.
- ii. Consider the following two-consumer, two-date, one-good exchange economy with two states of the world and consumption only at the second date. The consumers have the same probability beliefs $\pi = (\pi_1, \pi_2)$ about the states such that $\pi_s > 0$ for $s = 1, 2$ and $\pi_1 + \pi_2 = 1$. For each $i = 1, 2$, consumer i has endowment vector $\omega_i = (\omega_{1i}, \omega_{2i})$ and (Bernoulli) utility function $\pi_1 \log x_{1i} + \pi_2 \log x_{2i}$, where x_{si} is consumer i 's consumption in state s . Denote the aggregate endowments by $\bar{\omega} = (\bar{\omega}_1, \bar{\omega}_2)$, where $\bar{\omega}_s = \omega_{s1} + \omega_{s2}$ for $s = 1, 2$.

Assume that contingent commodity markets exist at the first date, and let p_1, p_2 denote the prices of the two contingent commodities. Compute p_1 / p_2 in Walrasian equilibrium and show that p_1 / p_2 is directly related to $\frac{\bar{\omega}_2}{\bar{\omega}_1}$ and to $\frac{\pi_1}{\pi_2}$. Provide some interpretation for this finding.

Consider a bilateral trading problem between one supplier and one potential consumer of some service. The supplier's cost per unit of supplying this service depends on the supplier's cost type, which is privately known only to the supplier. So when the supplier's type is θ , supplying q units of service for total pay x would give the supplier a net payoff $x - \theta q$.

The value of the q units of service to the buyer would also depend on the supplier's cost type θ according to the formula $\pi(q|\theta) = (4 + 2\theta)\sqrt{q}$. Thus, paying x to get q units from a supplier of type θ would give the buyer a net payoff $(4 + 2\theta)\sqrt{q} - x$.

Each individual could refuse to participate in trading, which would yield the result $q=0$ and $x=0$.

(a) Show a formula $\bar{q}(\theta)$ for computing the quantity of service that would yield the greatest possible sum of payoffs to the supplier and buyer if the supplier's type θ were commonly known.

Now in parts (b)-(f), assume that the supplier can misrepresent his type.

(b) Suppose that the supplier's type could be either $\theta_L=2$ or $\theta_H=3$. Let p_L denote the probability of the low-cost type θ_L , and let $p_H = 1 - p_L$ denote the probability of the high cost type.

Formulate the constrained optimization problem for finding a trading plan that maximizes the buyer's expected net payoff subject to incentive constraints and interim participation constraints.

(Note: $\pi(q|\theta_L) = 8\sqrt{q}$ and $\pi(q|\theta_H) = 10\sqrt{q}$.)

(c) Show formulas for explicitly computing the buyer's optimal incentive-compatible trading plan from part (b) in terms of the parameters p_L and p_H .

(d) Now consider the case where buyer believes that the supplier's cost type could be any number θ between 2 and 3, drawn from a Uniform probability distribution on this $[2,3]$ interval. Show formulas for computing the buyer's optimal incentive-compatible trading plan that maximizes the buyer's expected net profit subject to informational incentive constraints and interim participation constraints.

(You do not need to simplify these formulas as long as they could be used to explicitly compute all quantities and payments by anyone who can evaluate algebraic formulas and integrals.)

(e) Consider again the case where the supplier's type could only be either $\theta_L=2$ or $\theta_H=3$. Find the supplier's best safe trading plan (that is, find the trading plan that is best for both types of supplier among all incentive-compatible plans in which the buyer would get nonnegative profit from each type of supplier).

(f) For the situation in part (e), show that, if p_L is close enough to 0 and p_H is close enough to 1 then there exists a pooling plan that gives the buyer nonnegative expected profits (with these probabilistic beliefs) and is better for both types of supplier than the trading plan in (e).