Original

Harvard University
Department of Economics

General Examination in Microeconomic Theory

Fall 2004

1. You have **FOUR** hours.
2. Answer all questions

PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

Question for 2010

(1) Using a 3 or more period model, derive conditions under which it would be optimal for the government to ban an addictive drug.

(2) Give 5 reasons why it may not be optimal to allow the government to implement such a ban.

(3) Formally model 2 or these reasons. Hint: these reasons should not already be part of the model shown in part 1.

(4) Derive a number of comparative statics (at least 5) on when it is optimal for the government to be allowed to ban drugs and when it isn't.

for Part B (25 min)

Normal Form games:

(i) Define the concept of rationalizability for two person games in normal form.

(ii) Show that for all two person games in normal form the set of rationalizable equilibria is identical to the set of strategies that remain after iterated removal of strictly dominated strategies.

(iii) Show that any Nash equilibrium is survives iterated removal of strictly dominated strategies, and is therefore rationalizable by part (ii).

for Part B (35 min)

Extensive Form games:

(i) Give an example of a game in extensive form, with incomplete information, in which there are multiple equilibria. Make your example non-trivial, but still simple enough that you can find all the Nash equilibria.

(ii) Discuss several lines of game theoretic reasoning that enable us to say that some of equilibria are more plausible than others.

(iii) Use these lines of reasoning you have mentioned in part (ii) to distinguish among the equilibria in your example in part (i)-.

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Fall 2004

MICRO GENERALS

PROBLEM

Consider an Arrow-Debreu exchange economy with H agents ($h = 1, .., H$), two periods ($t = 0, 1$) and $S + 1$ date-events ($s = 0, .., S$). Each agent has endowment $\omega^h = (\omega_0^h, \omega_1^h, .., \omega_S^h)$ and state-independent utility functions

$$U^h(x_0, x_1, .., x_S) = u^h(x_0) + \beta \sum_{s=1}^{S} \gamma_s u^h(x_s).$$

The parameters $\beta, \gamma_1, .., \gamma_S$ are strictly positive, and each function u^h is strictly increasing, differentiable, and strictly concave. Let $\omega_s = \sum_{h=1}^{H} \omega_s^h$ denote the aggregate endowment in state $s \in \{1, ..., S\}$.

1. Assume that the aggregate endowment is the same in all future date-events: $\omega_s = \omega$ for all $s \in \{1, .., S\}$. Prove that if $(p, (x^h)_{h \in H})$ is an equilibrium, then

$$p_1/\gamma_1 = ... = p_S/\gamma_S$$

and $x_1^h = ... = x_S^h$ for all h.

2. We now assume that the aggregate endowment is random. Let $a, b \in \{1, .., S\}$. Show that

$$\frac{p_a}{\gamma_a} \geq \frac{p_b}{\gamma_b}$$

if and only if $\omega_b \geq \omega_a$.

for Part D (40 min)

In this problem there are n agents, each with a computer job to be run. These agents share a single computer which processes the jobs in sequence. It cannot begin one job until the previous one has been completed.

Each agent $i = 1,\ldots,n$ is characterized by two numbers: x_i is the length of the job to be run (in milliseconds) and c_i is the cost of waiting for the completion of its job (in cents per millisecond). In actual fact, these two parameters are the private information of the agents. The problem will be to elicit them in an incentive compatible manner and to arrange for the jobs to be sequenced in an efficient way.

(i) For the moment, however, assume that (x_i, c_i) are all known to a central agent. How should the jobs be sequenced so as to minimize the total waiting cost? Explain the economic principles you are using in words as well as in mathematics.

(a) First solve this problem for the special case where all the c_i are equal.

(b) Then look at the general case.

(ii) Returning now to the actual problem where the (x_i, c_i) are private information, assume that the agents can use monetary transfers t_i in a mechanism to help overcome incentive problems. Specifically, the payoff to agent i is

$$t_i - c_i \sum_{j \in \pi(i)} x_j$$

where $\pi(i)$ is the set of all agents whose jobs are processed before i's job.

Ignore the fact that $\sum t_i$ may not be zero. The social objective function is to minimize $\sum_i c_i \sum_{j \in \pi(i)} xj$ by the choice of the order of jobs. Can you use a Groves (also called a Clarke, or pivotal) mechanism to elicit a truthful response regarding (x_i, c_i) as a dominant strategy response from every agent?

(a) Explain the principles of Groves mechanisms in words.

(b) Then define the set of social decisions and apply these principles, describing the workings of Groves mechanism for this model mathematically.

(iii) Illustrate your answer to part (ii) by constructing a numerical example in a three person case. Assume that all the c_i are equal. Take $x_1 = 1$, $x_2 = 2$, $x_3 = 3$. How does your Groves mechanism distribute the total cost across the three players?

for Part D (20 min.)

Two people ($i = 1,2$) have the opportunity to establish a partnership that will share in an asset that generates a random amount of money x. The total to be shared is either -1 or y, with probability $.5$ for each. If they do not establish a partnership for this purpose, both receive 0.

The players' Bernoulli utility functions are given by

$$u_i(x_i) = -e^{-\alpha_i x_i}$$

The players know each other's Bernoulli utilities and can make any contingent contracts that specify the shares $x_1(x)$, $x_2(x)$ as a function of the total that is available $(x_1(x) + x_2(x) \equiv x)$.

(i) For which values of y will the players be able to find a contract that makes the formation of this partnership worthwhile.

(ii) Suppose that y satisfies the criterion that you found in part (i). The players decide to use the Nash Bargaining Solution to determine the optimal sharing rule. What is the realized expected utility for each of the players in this partnership as a function of y.