

Harvard University
Department of Economics

**Economics 2010b: Final Examination and
General Examination in Microeconomic Theory**

Fall 2003

1. You have **FOUR** hours.
2. Answer all questions

**PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE
QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.**

**PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.
PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.**

Consider the following simplified version of poker:

There are three cards: 1,2,3. There are two players ($i=1$ and $i=2$). The two players each contribute \$1 to the "pot" before play begins. Each player receives one card at random - the remaining card is discarded. Each player knows what card he has received but does not know which of the two other cards his opponent has received.

Player 1 moves first and has the option to "bet \$1", which means that player 1 adds \$1 to the pot.

Then player 2 has a move. If player 1 has bet, player 2 can match it, adding his \$1 to the pot. If player 2 does not match player 1's bet, player 1 takes the pot, which consists of \$1 of his own and \$1 contributed by player 2.

If player 1 does not bet, player 2 has the option to bet \$1 and if he does so, then player 1 has the option to match player 2's bet. As the roles of the players have been reversed, if the bet has not been matched player 2 takes the pot.

If any bets have been made and matched, or if no bets at all have been made, higher card takes all of what has been contributed to the pot.

- a) Describe the pure strategies of player 1.
- b) Does player 1 have any strictly dominated strategies?
- c) Show that player 1 has some weakly dominated strategies? Describe at least one of them.
- d) Show that, in a Nash equilibrium of this game, player 1 will in fact place a zero probability on the weakly dominated strategy that you have described in part c).
- e) For a general two person game, does the property of never placing any positive probability on a weakly dominated strategy characterize all the Nash equilibria of the game?
- f) Demonstrate that the following describes a pair of equilibrium strategies: Player 1 never bets in the first round except when holding a "2", and in this case bets with probability $1/3$. Player 2 bets $1/3$ of the time when holding a 1, if player 1 has not made a prior bet. Player 2 bets $1/3$ of the time when holding a 2, if player 1 has made a prior bet.
- g) What is the value of the game to the two players?

This question concerns adverse selection and the demand for life insurance.

A group of consumers has income y_0 in period one. In period two they will have no income. Each consumer knows his or her probability of death, π_i . Insurance companies can observe nothing about the consumer at all – neither personal characteristics nor any decisions that the consumer makes.

Insurance companies quote a price per unit of insurance p . If a consumer chooses to buy x units of insurance, the company collect px in the current period and pays out x in the event that the consumer dies.

The consumer makes two choices: how much insurance x to buy, and how much to save for the future, s . Any savings that the consumer makes is added to future income if he or she is alive, but insurance is added to future income only if the consumer is dead.

There are many consumers. They care about their income in each period and life/death condition because their primary concern is for the consumption available to other members of their family.

All have the same Bernoulli utility functions. They may differ with respect to their incomes and their death probabilities.

a) How are the informational assumptions of this problem different from that in typical adverse selection models?

b) For the case in which the Bernoulli utilities are all logarithmic, that is:

$$u(c_1, c_{2a}, c_{2d}) = \log c_1 + (1 - \pi_i) \log c_{2a} + \pi_i \log c_{2d}$$

derive a formula for the elasticity of the demand for life insurance with respect to its price.

c) Suppose that competition among insurance companies means that the expected payments to individuals must be equal to the premiums collected. Show that p is greater than the average π_i in the population.

d) Show that, for distribution of π in the population, this market will not unravel completely? That is, why is it the case that (unlike in the typical adverse selection model), there must be a price p at which there are some purchases of insurance?

A country has L units of labor living within its borders. There are two consumption goods, A and B. To produce one unit of A requires $\frac{1}{2}$ unit of labor; to produce one unit of B requires 2 units of labor. There are constant returns to scale.

- (a) If the wage is w , what are the prices of A and B?
- (b) If consumers have Cobb-Douglas preferences, how much A and B are produced?

Now suppose that a new production method is invented under which one unit of A can be made from $\frac{1}{8}$ unit of labor combined with $\frac{1}{4}$ unit of B. Assume that all production is in the form of “flows” so that the flow output of B used in the production of A could immediately be transferred there to produce a flow output of A.

- (c) Will this new technique be adopted? Explain.

Now assume that there is a “mirror-image” country which has L' units of labor. In this country the technological relationships between A and B are exactly reversed so that A requires 2 units of labor and B requires $\frac{1}{2}$ unit of labor. Moreover, the newly invented technique is in the B industry, where a unit of B requires a flow of $\frac{1}{8}$ unit of labor together with $\frac{1}{4}$ of A, per unit flow output of B.

- (d) Suppose that these countries can engage in international trade in A and B, but that labor is immobile across national boundaries. Which production methods are used? What are the prices?
- (e) Is everyone better off as a result of international trade? Explain.
- (f) Is your result in e) necessarily true, or does it just happen to be true for the particular forms of technology, preferences and labor endowments in this example?

a. Consider a world populated by a large number of firms and workers. All firms are identical, workers differ in their ability level. The ability, a , is distributed in the population uniformly on the interval $[0,1]$, $a \sim U[0,1]$. If a worker of ability a remains self employed his output is a^2 . If a worker of ability a is employed by a firm his output equals his ability a . Assume that the type of each worker is his private information, describe all competitive equilibria in the market with adverse selection. Now assume that a worker's productivity if employed by a firm is $2a$. Describe all competitive equilibria in this case.

b. Suppose the set of alternatives X is finite and contains at least three alternatives. Suppose each agent has a strict preference ordering over set X (any preferences are allowed). Suppose SCF, f is truthfully implementable in dominant strategies and that if an alternative is ranked first by all agents then it is chosen according to f . Does such SCF exist? If so can you describe all such SCF, if not explain why such SCF may not exist. (To receive full credit you must state theorems that you use and show that conditions of these theorems are satisfied.)