

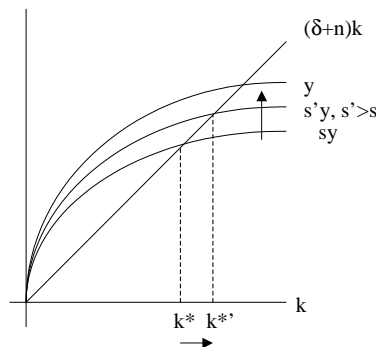
# 1 Macro Question 1

## 1.1 (1 point)

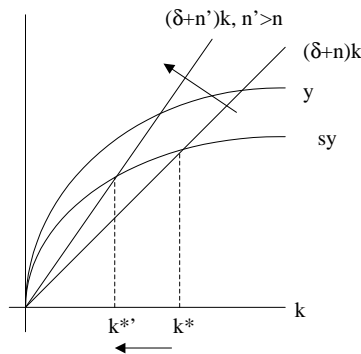
$$\Delta k = sk^\alpha - (\delta + n)k$$

## 1.2 (2 points)

The Solow model shows that the saving rate,  $s$ , is a key determinant of  $k^*$ . If  $s$  is high, the economy will have a large capital stock and a high level of output. If  $s$  is low, the economy will have a small capital stock and a low level of output.



Population growth reduces the accumulation of capital per worker by spreading the capital stock more thinly among a larger population of workers.



## 1.3 (3 points)

Whether economies converge depends on why they differed in the first place. On the one hand, if two economies with the same steady state (the OECD countries, for example) happened to start off with different capital stock, then we should expect them to converge. The economy with the smaller capital stock will naturally grow more quickly. On the other hand, if two economies have different steady states (our broad group of countries, for example), perhaps

because the economies have different rates of saving, then we should not expect convergence. Instead, each economy will approach its own steady state.

The pattern that a lower capital stock predicts a higher growth, conditional on  $k^*$ , is called conditional convergence. (In contrast, the prediction that a lower capital stock raises  $\Delta k/k$  without any conditioning is called absolute convergence).

#### 1.4 (1 point)

If labor is paid its marginal product (MPL), then the wage rate ( $w$ ) will be given by

$$w = MPL = \frac{\partial Y}{\partial L} = (1 - \alpha) K^\alpha L^{1-\alpha-1} = (1 - \alpha) \frac{K^\alpha L^{1-\alpha}}{L} = (1 - \alpha) \frac{Y}{L}$$

Then, employees' income,  $wL$ , is  $(1 - \alpha)Y$ . In words, employees receive fraction  $(1 - \alpha)$  of national income. Therefore, the implied  $\alpha$  for the US is  $\frac{1}{3}$ .

#### 1.5 (3 points)

Under the neoclassical framework, production factors are paid their marginal products. Thus, the wage rate,  $w$ , is given by

$$w = MPL = \frac{\partial Y}{\partial L} = (1 - \alpha) K^\alpha L^{-\alpha} = (1 - \alpha) \frac{K^\alpha}{L^\alpha} = (1 - \alpha) k^\alpha$$

Similarly for the rental price of capital,  $r$

$$r = MPK = \frac{\partial Y}{\partial K} = \alpha K^{\alpha-1} L^{1-\alpha} = \alpha \frac{L^{1-\alpha}}{K^{1-\alpha}} = \frac{\alpha}{k^{1-\alpha}}$$

It is not hard to see that  $w$  goes down and  $r$  goes up with  $L$ . Thus, we should expect lower wages and higher rental prices of capital as a result of the immigration from Mexico to the US. Intuitively, a larger labor force makes labor cheaper. Also, it is not unreasonable to think that a given stock of capital can produce more when it is operated by a larger number of workers.

If we assume that the US was at its steady state before the migration, it will return to its initial state over the long run. Thus, the values of both  $w$  and  $r$  will be restored in the long run too.

## 2 Macro Question 2

### 2.1

#### 2.1.1 (1 point)

Job separations equal the fraction of employed people who lose their jobs:  $sE$ . Similarly, job finding equals the fraction of unemployed people who find a job:  $fU$ . In equilibrium,  $sE = fU$ .  $L = E + U$  implies

$$1 = \frac{E+U}{L} = \frac{E}{L} + u$$

Then

$$s \frac{E}{L} = s(1 - u) = fu = f \frac{U}{L}$$
$$u^* = \frac{s}{f+s} = \frac{1}{\frac{f}{s} + 1}$$

### 2.1.2 (2 points)

On the one hand, UI may reduce the urgency of finding work. Hence, it may reduce  $f$ . On the other hand, by allowing workers more time to search, UI may lead to better matches between jobs and workers, which could lead to a lower  $s$ . Thus, the effect of unemployment insurance on the natural rate of unemployment need not be certain. Thus, the equation in (a) suggests that the effect on the natural rate of unemployment of UI need not be certain: with both  $f$  and  $s$  falling, the net change in  $\frac{f}{s}$  is unknown in principle.

## 2.2 (2 points)

In equilibrium money demand equals money supply

$$L(r, Y) = \frac{M}{P}$$

In the short run  $P$  is fixed. Therefore, if money supply is not under the control of the Fed, the shock could be offset by decreasing the opportunity cost of holding money, encouraging households to hold it. Hence, the Fed should lower interest rates.

## 2.3 .

### 2.3.1 (1 point)

Taxes reduce disposable income in the following way:

$$C_1 + \frac{C_2}{1+r} = (Y_1 - T_1) + \frac{(Y_2 - T_2)}{1+r} = \left( Y_1 + \frac{Y_2}{1+r} \right) - \left( T_1 + \frac{T_2}{1+r} \right)$$

### 2.3.2 (1 point)

The government obeys the following budget constraint:

$$G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r}$$

Thus, for a *Ricardian* household the effective budget line is

$$C_1 + \frac{C_2}{1+r} = \left( Y_1 + \frac{Y_2}{1+r} \right) - \left( G_1 + \frac{G_2}{1+r} \right)$$

This expression clearly shows that if government expenditures stay the same, there will be no income effects associated with the current tax cut: the higher current disposable-income will be compensated by a lower future disposable-income. In principle we have no reasons to believe that there are interest rate changes associated with the stimulating plan. Thus, we do not expect substitution effects either. As a result, current consumption is expected to stay the same.

### 2.3.3 (1 point)

If current government expenditures increase but future public consumption stays the same, the government budget line shows that the present value of taxes must be higher. Then, we will have a negative income effect on current consumption, regardless of the tax path. (Again, we are assuming that interest rates are not affected by the fiscal stimulus, so that we do not have any substitution effects on current consumption).

### 2.3.4 (2 points)

Why might Ricardian equivalence fail—meaning that current tax cuts do affect C? People might spend even if future taxes are higher because:

1. Shortsightedness, lure of immediate gratification.
2. Tax cuts relax borrowing constraints.
3. Taxes may fall on future generations (whom you don't care about!). (Barro argues that people do care about their children, so will save to give them larger bequests).
4. “Starve the Beast”: tax cut today may imply lower government expenditures, so that there is a true income effect.
5. Actual taxes distortionary: distortionary taxes affect incentives to save, work, invest...

## 3 Macro Question 3

### 3.1 (2 points)

Let us start with

$$Y_0 \equiv AK_0 + BK_0^\alpha L_0^{1-\alpha}$$

If we double the amount of capital and labor, output will double too:

$$\begin{aligned} Y_1 &\equiv A(2K_0) + B(2K_0)^\alpha (2L_0)^{1-\alpha} = 2(AK_0) + B2^\alpha 2^{1-\alpha} K_0^\alpha L_0^{1-\alpha} \\ &= 2(AK_0) + B2^{\alpha+1-\alpha} K_0^\alpha L_0^{1-\alpha} = 2(AK_0 + BK_0^\alpha L_0^{1-\alpha}) = 2Y_0 \end{aligned}$$

Then, this technology does exhibit constant returns to scale. Regarding returns to capital, this production function exhibits a diminishing marginal product of capital:

$$\begin{aligned} Y_2 &\equiv A(2K_0) + B(2K_0)^\alpha L_0^{1-\alpha} = A(2K_0) + 2^\alpha BK_0^\alpha L_0^{1-\alpha} \\ &< 2(AK_0) + 2BK_0^\alpha L_0^{1-\alpha} = 2(AK_0 + BK_0^\alpha L_0^{1-\alpha}) = 2Y_0 \end{aligned}$$

Another way to see this is by looking directly at the equation for the marginal product of capital:

$$MPK = \frac{\partial Y}{\partial K} = A + \alpha BK^{\alpha-1}L^{1-\alpha} = A + \alpha B \frac{L^{1-\alpha}}{K^{1-\alpha}}$$

Note that, unlike the *conventional* case,  $MPK$  does not become zero for an arbitrarily large  $K$ , but  $A$ , which is greater than zero. Thus, more capital leads to more output always .

### 3.2 (2 points)

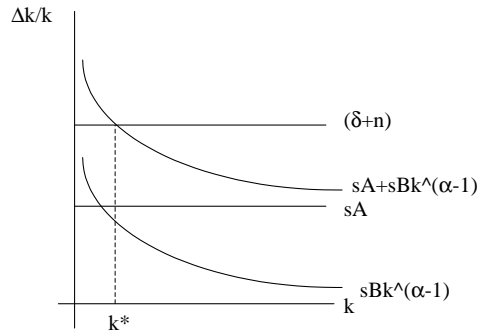
$$\Delta k = sy - (\delta + n)k = s(Ak + Bk^\alpha) - (\delta + n)k$$

Thus

$$\frac{\Delta k}{k} = s(A + Bk^{\alpha-1}) - (\delta + n) = sB \frac{1}{k^{1-\alpha}} + (sA - \delta - n)$$

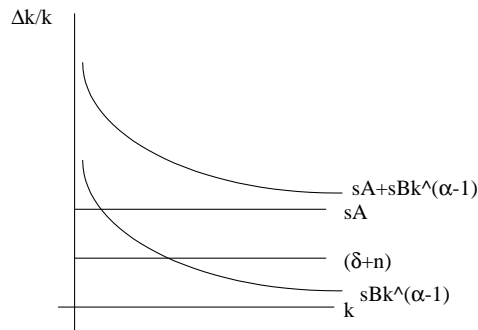
### 3.3 (2 points)

If  $sA < \delta + n$ , there is a positive level of  $k$  for which  $\frac{\Delta k}{k}$  is zero:  $k^{1-\alpha} = \frac{sB}{\delta+n-sA} > 0$ . Then, this model does not predict long-run growth, but convergence:



### 3.4 (2 points)

If  $sA > \delta + n$ , then  $\frac{\Delta k}{k}$  is always positive:  $\frac{\Delta k}{k} = sB \frac{1}{k^{1-\alpha}} + (sA - \delta - n) > (sA - \delta - n) > 0$ . Then, this model does predict long-run growth (and no convergence):



### 3.5 (2 points)

When  $\alpha = 1$ ,  $\frac{\Delta k}{k} = sB + (sA - \delta - n)$ . From (c) we know that  $(sA - \delta - n) < 0$ . However,  $sB > 0$ . Hence, in this case the model predicts a constant long-run growth, but there is not enough information to determine whether it is positive, negative, or zero.

## 4 Macro Question 4

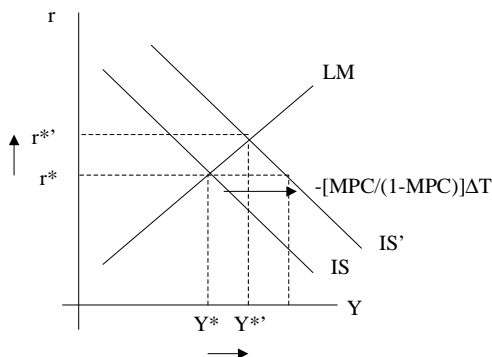
### 4.1 (4 points)

The current tax cut will increase disposable income, thus shifting the IS curve to the right. Moreover, the magnitude of the shift will be given by

$$-\frac{MPC}{1-MPC} \Delta T$$

where MPC is the marginal propensity to consume. ( $-\frac{MPC}{1-MPC}$  is known as the tax multiplier).

At the new equilibrium, both income (Y) and the interest rate (r) will be higher:



Disposable income goes up because income is higher and taxes are lower. Hence, there is a *double* positive income effect on consumption. On the other hand, a higher interest rate leads to a lower investment.

### 4.2 (4 points)

The intertemporal budget constraint for households is given by

$$C_1 + \frac{C_2}{1+r} = \left( Y_1 + \frac{Y_2}{1+r} \right) - \left( T_1 + \frac{T_2}{1+r} \right)$$

But the government also obeys a budget constraint:

$$G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r}$$

Thus, for a forward-looking household the effective budget line is

$$C_1 + \frac{C_2}{1+r} = \left( Y_1 + \frac{Y_2}{1+r} \right) - \left( G_1 + \frac{G_2}{1+r} \right)$$

Thus, if the household knows that the present and future value of government expenditures do not change, he will infer that the current tax cut will have to be paid back in the future. Hence, there will be no income effects associated with the tax cut, and the household will not spend it, but save it. In the diagram above, the IS curve will stay the same, and so will do the variables in (a). This result is known as Ricardian equivalence. (Among other things, we are implicitly assuming that taxes are lump-sum, so that they are not distortionary).

### **4.3 (2 points)**

Tax cuts may affect consumption when Ricardian equivalence does not hold. What might cause Ricardian equivalence to fail? Among others, borrowing constraints: a tax cut may relax borrowing constraints, inducing households to spend the tax cuts.