Harvard University
Department of Economics

General Examination in Macroeconomic Theory

Fall 2003

You have **FOUR** hours.

Solve all questions.

The exam has 4 parts. Each part has its own sheet.
Please spend the following time on each part.

I.	60 minutes
II.	60 minutes
III.	60 minutes
IV.	60 minutes

PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

Question 1 - 60 minutes (60 points) Fall 2003

Part 1: Answer both of the following two questions:

1. If a central bank carries out monetary policy by targeting a specific inflation rate (as many central banks outside the United States now do), how can an economist observing its actions, and also observing the behavior of the economy, tell whether "inflation targeting" in this case means (a) that the central bank has preferences only with respect to inflation or (b) that the central bank has preferences with respect to both inflation and real output? If the central bank does have preferences with respect to both inflation and real output, how can the economist infer the relative weights that it places on these two variables? Be as explicit as you can in explaining the reasoning behind your answers, and in stating any underlying assumptions.

2. "Indexing taxes, debt contracts and other economic arrangements so as to protect citizens from inflation is a bad idea. All that will happen is that the central bank will feel free to create more inflation than before, and in the end everyone will be worse off despite the added protection."

   Is the perverse outcome described in this statement plausible? Why or why not?

Question 2 - 60 minutes (60 points)

Fall 2003

Problem based on Bertola and Caballero 1990: Consider a firm that faces a decision problem. The firm tries to keep an Ito process, $X(t)$, near zero. Specifically, the firm tries to maximize the net present value of instantaneous payoffs

$$-\frac{b}{2}X^2$$

where the discount rate is ρ. Assume $X(t)$ is described by

$$dX = \alpha dt + \sigma dz + (\text{discrete adjustments undertaken by the firm}).$$

A discrete upward adjustment of I units costs the firm $C_u + c_u I$, with $C_u \geq 0$ and $c_u \geq 0$. A discrete downward adjustment of $|I|$ units costs the firm $C_d + c_d|I|$, with $C_d \geq 0$ and $c_d \geq 0$. Solving this problem, is equivalent to finding four endogenous boundaries for $X(t)$: U, u, d, D. When $X(t)$ reaches U, the firm discretely raises $X(t)$, jumping up to $X(t) = u$. When $X(t)$ reaches D, the firm discretely lowers $X(t)$, jumping down to $X(t) = d$.

a. Economically motivate this problem. Specifically, find a convincing economic problem that is well-described by the assumptions summarized above.

b. Intuitively explain why $u \leq d$. Under what conditions will this inequality hold strictly? When will $u = d$. Explain your reasoning with intuition. Intuitively explain why $U \leq u$ and $d \leq D$. Under what conditions will these inequalities hold strictly? When will $u = U$? When will $d = D$? Explain your reasoning with intuition.

c. Derive the continuous-time Bellman Equation in the continuation region (i.e., for $U \leq X(t) \leq D$). Apply Ito's Lemma to show that in the continuation region

$$\rho V(X) = -\frac{b}{2}X^2 + \alpha V'(X) + \frac{1}{2}\sigma^2 V''(X) \tag{1}$$

Why does $\frac{\partial V}{\partial t}$ not appear in this equation? Interpret this equation as a decomposition of the required return.

d. Using the fact that

$$V(X) = -\frac{b}{2}\left(\frac{X^2}{\rho} + \frac{\sigma^2 + 2\alpha X}{\rho^2} + \frac{2\alpha^2}{\rho^3}\right)$$

is a particular solution to Equation 1, show that the general solution to Equation 1 is

$$V(X) = -\frac{b}{2}\left(\frac{X^2}{\rho} + \frac{\sigma^2 + 2\alpha X}{\rho^2} + \frac{2\alpha^2}{\rho^3}\right) + A_1 e^{\alpha_1 X} + A_2 e^{\alpha_2 X}$$

with roots

$$\alpha_1 = \frac{-\alpha + \sqrt{\alpha^2 + 2\sigma^2\rho}}{\sigma^2}$$

$$\alpha_2 = \frac{-\alpha - \sqrt{\alpha^2 + 2\sigma^2\rho}}{\sigma^2}.$$

e. Write down a six equation system that you would use to solve for the unknown variables that characterize the firm's value function. Intuitively explain how to generate all of the equations. Why do we need six equations? (Do not try to solve the system.)

f. Is it possible to parameterize the model in a way that makes U greater than 0? How about u greater than 0? Explain intuitively.

Capital- and Labor-Augmenting Technological Progress (60 minutes)

Assume that the production function takes the form

$$Y = F(AK, BL),$$

where A represents capital-augmenting technology and B represents labor-augmenting technology. Assume that A and B are each growing exogenously at constant rates. The function F is a usual neoclassical one, so that output satisfies constant returns to scale in the two inputs, K and L.

1. Use the condition of constant returns to scale to prove Euler's Theorem:

$$Y = (\partial Y/\partial K)\cdot K + (\partial Y/\partial L)\cdot L.$$

2. Differentiate the production function with respect to time to work out a formula for the growth rate of output per worker, $y = Y/L$. Using the result in 1., the solution should involve only the growth rate of capital per worker, $k = K/L$, the growth rates of A and B, and the term $(\partial Y/\partial K)\cdot(K/Y)$. What does this last term represent?

Suppose that we are looking for a steady-state growth situation in which y is growing at a constant rate and k is growing at a constant rate, possibly different from the growth rate of y. Assume that the term $(\partial Y/\partial K)\cdot(K/Y)$ is not always constant—that is, it depends on the ratio of AK to BL. In this case,

3. What condition has to hold for the growth rate of k in order for the growth rate of y to be constant? What is the corresponding growth rate of y? Does it necessarily equal the growth rate of k?

4. Suppose that the two factors are paid their marginal products, so that the rental price is $R = \partial Y/\partial K$, and the wage rate is $w = \partial Y/\partial L$. Given the result in 3., how do the values of R and w evolve over time? If we are looking for a solution in which R is constant in the steady state, whereas w is rising, what conditions have to hold for A and B? What does this mean about capital- and labor-augmenting technological progress?

5. If the term $(\partial Y/\partial K)\cdot(K/Y)$ is always constant, what form does the production function take? How do the results change in this case?

This is a question about a model that has been proposed to interpret the relationship between elections and key macroeconomic policies.

A.1) In every period the government provides total services g (an exogenous variable throughout this model). It has two tax instruments to do so, and the total revenues from the two instruments are τ and π. The government chooses both τ and π. A certain amount ξ of the public good is produced by the policy-maker "for free", i.e. without the need to pay for it with taxes. There is no government borrowing or lending. What is the government's flow budget constraint?

A.2) How do opportunistic models of the political business cycle interpret ξ?

A.3) Suppose that voters' per-period utility is given by

$$U = -\tau - \pi - h(\pi) + \eta,$$

where $h(0) = 0$, and $h' > 0$, and η is a "taste shock" that determines how well voters like the policy-maker currently in office. How would you interpret the first three terms, and what do they tell you about the difference between τ and π?

A.4) What would a benevolent dictator do in this economy?

Elected policy-makers spend two periods in office. During the second period of a policy-maker's term in office, elections are held to determine whether the current policy-maker is re-elected, or if a challenger will serve in the next two periods. Under a certain set of assumptions (on the stochastic processes for ξ and η, on the sequence of events during an election year, on voters' and candidates' beliefs and objectives, and on whom observes what when) it is possible to show that a separating equilibrium exists in which some incumbents set $\pi > 0$.

A.4) What is the intuition for this result?

A.5) State all the assumptions that are needed to obtain this result. Explain very clearly and very explicitly why each of these assumptions is crucial.

A.6) What does existing empirical evidence say in support or against this theory?