

## The PPP Puzzle

- Reversion of the RER to parity much too slow. Much too much persistence in RER.
- Standard Metric is the half-life, i.e. time it takes for half the effects of a given shock to dissipate.
- Rogoff (1996) talks of a “consensus view” of a half-life between 3-5 years. Much too long to be compatible with arbitrage. Questions very validity of PPP in short to medium run.
- Alternative views: market power and pricing to market, non-traded local costs...
- But Engel (1999): most of the puzzle originates in traded goods, where presumably alternative explanations are hardest.
- Furthermore, Chari-Kehoe-McGratten (2002) show model with plausible rigidities cannot reproduce this type of RER persistence.

## Econometric Debate

- Longer data sets afford more power to reject unit roots. Frankel (1990), Lothian-Taylor (1996), Diebold-Husted-Rush (1991)
- Panels too. Frankel-Rose (1996), Oh (1996), Lothian (1997)
- Non-linearities as well, i.e. “adjustment bands”. Michael-Nobay-Peel (1997), Taylor-Peel-Sarno (2001), Kilian-Taylor (2002).
- On the other hand, Murray-Pappell (2002 x 3) argue is an attenuating bias in half-life estimates (because of short samples and very persistent processes). PPP Puzzle is worse than you think.
- Verdict unclear. Kilian and Zha (2002) survey international economists, and find confirmation of “consensus view”, between 3 and 5 years...

## This paper

- Points to a quantitatively (very) important cross-sectional aggregation bias.
- There is every theoretical reason to expect the *components* of CPI-based RER to have very different persistence properties.
- E.g. transaction costs, tariffs, market structure, local costs... all should differ by goods (sector).

We show:

1. failure to correct for this heterogeneity results in a bias (Pesaran-Smith 1995)
2. the bias is positive
3. uncorrected estimates are in line with the “consensus view” (as befits since our data is standard)
4. corrected estimates are in line with plausible rigidities, at 14 months. PPP puzzle is no more.
5. aggregation bias can explain why traded goods contribute so much to RER dynamics. Because their dynamics are more heterogeneous.

# Plan

Data

Aggregation Bias in Theory

Aggregation Bias in Practice: PPP Strikes Back

Robustness and Alternative Explanations

Traded vs. Non-Traded Goods

Conclusions

# Data

International Sectoral Dataset – Eurostat non-harmonized price indices

Price (indices) data for 19 (two-digit) sectors across 13 countries

Monthly data 1975:1 – 1996:12

Belgium, Denmark, Germany, Greece, Spain, France, Ireland, Italy, the Netherlands, Norway, Portugal, Finland, UK, US.

Bread, Meat, Dairy, Fruits, Tobacco, Drinks, Clothing, Footwear, Rents, Fuel, Furniture, Domestic Appliances, Vehicles, Public Transportation, Communication, Sound, Leisure, Books, Hotels.

And the corresponding aggregate indices (Also used IFS standard CPI indices, with identical results.)

Relative prices are defined as:

$$q_{i,j,t} = \ln \left( \frac{e_{jt} \cdot P_{i,j,t}}{P_{i,US,t}} \right)$$

where  $i$  denotes sector,  $j$  country and US is anchor

## Stationarity

Panel results clear:

Non-Stationarity rejected for both relative prices and RER.

**Table 1: Unit Root Tests**

Test	Trend	$\ln \left( \frac{(P_i * e_{i,us})}{P_{us}} \right)$	$e_{i,us}$	$\ln \left( \frac{(P_{ij} * e_{i,us})}{P_{ijs}} \right)$
IPS ADF	no	1.995 [0.023]	-2.397 [0.008]	-5.915 [0.000]
IPS ADF	yes	2.505 [0.006]	0.334 [0.369]	-4.218 [0.000]
LL	no	-2.030 [0.021]	-1.800 [0.036]	-16.901 [0.000]
LL	yes	-1.984 [0.024]	-1.773 [0.038]	-16.422 [0.000]
LL <sup>1</sup>	yes	-11.902 [0.000]	-8.092 [0.000]	-7.011 [0.000]

Similar to Lothian (1997) across countries.

Even stronger evidence across sectors.

## Aggregation Bias in Theory: Two-sector Case

$$x_{1t} = \theta_1 x_{1t-1} + e_{1t}$$

$$x_{2t} = \theta_2 x_{2t-1} + e_{2t}$$

with  $E(e_{1t}e_{2t}) = 0$ ,  $E(e_{it}e_{is}) = 0$  for  $s \neq t$ ,  
 $E(e_{it}e_{it}) = \sigma^2$  and  $E(e_i) = 0$ .

$$\begin{aligned} x_t &= \theta_1 \frac{x_{1t-1}}{2} + \theta_2 \frac{x_{2t-1}}{2} + e_t \\ &= \left( \bar{\theta} + \frac{\theta_1 - \theta_2}{2} \right) x_{t-1} - \frac{\theta_1 - \theta_2}{2} x_{2t-1} + e_t \end{aligned}$$

where  $\bar{\theta} = \left( \frac{\theta_1 + \theta_2}{2} \right)$  and  $e_t = \frac{e_{1t} + e_{2t}}{2}$ .

The OLS estimate is  $\hat{\theta} = E(x_t x_{t-1}) / \sigma_x^2 \Rightarrow$

$$\hat{\theta} = \bar{\theta} + \frac{\theta_1 - \theta_2}{2} \left( 1 - \frac{\sigma_{x_2}^2}{2\sigma_x^2} \right)$$

where  $\sigma_x^2 = E(x_t x_t)$  and  $\sigma_{x_2}^2 = E(x_{2t} x_{2t})$

With  $0 < \theta_i < 1$  and  $\theta_1 > \theta_2$ , it is easy to see that  $\hat{\theta} > \bar{\theta}$ .

Define  $\Delta = 1 - \frac{\sigma_{x_2}^2}{2\sigma_x^2} > 0$ . The OLS aggregate estimate can be rewritten

$$\hat{\theta} = \theta_1 \frac{1 + \Delta}{2} + \theta_2 \frac{1 - \Delta}{2}$$

- Therefore OLS gives a larger weight to the more persistent component of the real exchange rate, resulting in apparently large aggregate persistence.
- The bias increases with the discrepancy between  $\theta_1$  and  $\theta_2$ , i.e. with the extent of sectoral heterogeneity.

## Aggregation Bias in Theory

Suppose true model is:

$$q_{it} = \alpha_i + \lambda_i q_{it-1} + \varepsilon_{it}$$

with

$$\lambda_i = \lambda + \eta_i$$

AR(1) not crucial to the argument. Point is (sectoral) heterogeneity.

Then pooled estimates will be biased (Pesaran-Smith 1995), since:

$$q_{it} = \alpha_i + \lambda q_{it-1} + v_{it}$$

$$v_{it} = \varepsilon_{it} + \eta_i q_{it-1}$$

Residuals unavoidably correlated with independent variable. IV will be impossible since need for instrument highly correlated with regressor and orthogonal to residuals.

Paper shows that

$$\widehat{\lambda} - \lambda = \frac{E\left(\frac{\eta_i}{1-\lambda_i^2}\right)}{E\left(\frac{1}{1-\lambda_i^2}\right)}$$

where  $\widehat{\lambda}$  denotes the probability limit of the fixed-effects estimator of  $\lambda$ .

Also shows that the bias is positive as soon as  $0 < \lambda_i < 1$ , i.e. for positive serial correlation in sectoral relative prices.

(This happens as sectors with positive  $\eta_i$  have higher weights than negative ones in the above expression)

Expression shows that bias increases with extent of heterogeneity.

So pooled estimator is biased positively, to an extent that increases with heterogeneity.

## Proof that bias is positive

For  $0 < \lambda_i < 1$ ,

$$E\left(\frac{\eta_i}{1 - \lambda_i^2}\right) = \int_{-\lambda}^0 \frac{\eta_i}{1 - (\eta_i + \lambda)^2} f(\eta_i) d\eta_i + \int_0^{1-\lambda} \frac{\eta_i}{1 - (\eta_i + \lambda)^2} f(\eta_i) d\eta_i$$

For all  $\eta_i < 0$ ,  $1 - (\eta_i + \lambda)^2 > 1 - \lambda^2$ ; conversely for all  $\eta_i > 0$ ,  $1 - (\eta_i + \lambda)^2 < 1 - \lambda^2$ . Hence all the positive  $\eta_i$  have higher weights than the negative ones in the above expression. Since in  $E(\eta_i)$  they have equal weights and since  $E(\eta_i) = 0$ , it follows that  $E\left(\frac{\eta_i}{1 - \lambda_i^2}\right) > 0$ . QED.

# Aggregation

Argument exactly identical for aggregate estimators: RER is a (weighted) average of sector-specific relative prices. Consider a simplified real exchange rate:

$$q_t = \frac{1}{N} \sum_{i=1}^N q_{i,t}$$

Then residual in an AR(1) that does not account for heterogeneity writes

$$e_t = \overline{\varepsilon_{i,t}} + \frac{1}{N} \sum_{i=1}^N \nu_i q_{i,t-1}$$

Should be clear generalizes to AR(p) and any weighting device. Same positive bias.

## Lagged Dependent Variables

Problem compounded in dynamic estimations with lagged dependent variables (LDV).

RER estimations typically run with Fixed Effects, i.e. first differencing

$$q_{i,j,t} = \alpha_{i,j} + \sum_{p=1}^P \gamma_p q_{i,j,t-p} + \varepsilon_{i,j,t}$$

Well-known problem is correlation between (first-differenced) LDV and (first-differenced) residuals. Instrument LDV with its own lagged levels (Anderson-Hsiao or Arellano-Bond)

But, with heterogeneity uncorrected for, IV won't do any good at all, since residuals will remain correlated with **any** lagged level of the LDV (through  $\eta_i$ )

## Random Coefficients Model

Unbiased estimator should allow for parameter heterogeneity, i.e.

$$q_{i,j,t} = \alpha_{i,j} + \sum_{p=1}^P \gamma_{i,j,p} q_{i,j,t-p} + \varepsilon_{i,j,t}$$

with

$$\gamma_{i,j,p} = \gamma_p + \nu_{i,j}^1 \text{ and } \alpha_{i,j} = \alpha + \nu_{i,j}^2.$$

Hildreth-Houck (1968) introduced Random Coefficients Model, which uses GLS to account for the sector-specific noise in both intercept and coefficients.

We also implement (Swami and Hausman) tests to verify the relevance of parameter heterogeneity in the data.

# Aggregation Bias in Practice

Can we reproduce existing “consensus view”?

**Table 2: Half Life Estimates using Aggregate Data**

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$$q_{j,t} = \alpha_j + \sum_{p=1}^P \rho_p q_{j,t-p} + \nu_{j,t}$$


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Model	$P$	$\sum_{p=1}^P \rho_p$	Half Life	Confidence Interval
OLS	12	0.9998	5589	44,∞
Fixed Effects	32	0.9733	39	14,58
Anderson Hsiao	36	0.9905	108	68,∞
Arellano Bond	12	0.9689	27	6,29
<sup>a</sup> $H0 : \alpha_j = 0$	3.04 (0.0001)			
<sup>b</sup> $H0 : E(\alpha_j, X) = 0$	34.33 (0.0006)			
<sup>c</sup> $H0 : \rho_j = \rho$	111.38 (0.999)			
<sup>d</sup> $H0 : \rho_j = \rho$	2.45 (0.1175)			

Notes: The panel is based on aggregate relative prices indices for 13 countries over the period 1975:1 - 1996-12. The choice of P is based on general to specific lag selection procedure with a maximum lag of 36 for all models, except Arellano and Bond where the maximum is 12. For the GMM estimator five lags of the levels of relative prices were used as instruments. The confidence intervals for the half life estimates were estimated using bootstrap with 1500 replications for all specifications, except Arellano and Bond where they were restricted to 500. “a” is a test for fixed effects, “b” is the Hausman test, “c” is a Swami test for coefficient homogeneity, while “d” denotes a Hausman type test for homogeneity.

Parameter homogeneity is not rejected (but across countries!). Lower end of the “consensus” (2-3 years)

# PPP Strikes Back

**Table 3: Half Life Estimates using Disaggregate Data**

$$q_{i,j,t} = \alpha_i + \sum_{p=1}^P \gamma_{i,p} q_{i,j,t-p} + \varepsilon_{i,j,t}$$

Model	$P$	$\sum_{j=1}^P \rho_j$	Half Life	Confidence Interval
RCM	5	0.9481	14	5,24
OLS	12	0.9996	2063	9, $\infty$
Fixed Effects	12	0.9698	27	7,42
Anderson Hsiao	7	0.9974	281	1, $\infty$
$^a H0 : \alpha_i = 0$	1.85 (0.000)			
$^b H0 : E(\alpha_i, X) = 0$	1196.32 (0.000)			
$^c H0 : \beta_i = \beta$	2317.37 (0.000)			
$^d H0 : \beta_i = \beta$	32.68 (0.000)			

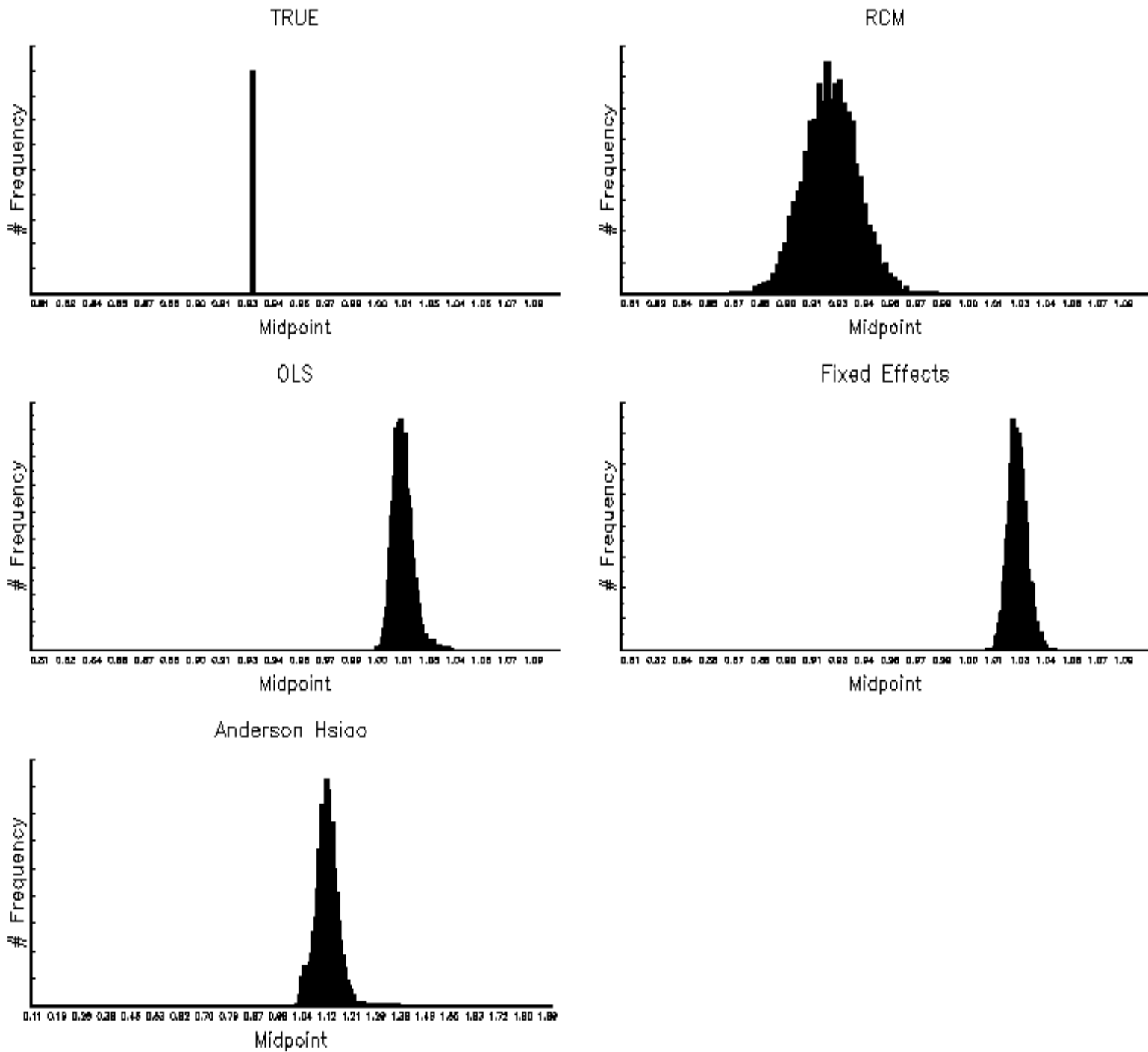
Notes: The estimates are based on a panel of relative prices for 19 goods from 13 countries over the period 1975:1 - 1996:12. The lag length  $P$  was chosen via a general to specific lag selection procedure with a maximum lag of 12 for all models. The confidence intervals for the half life estimates are based on 1500 bootstrap replications for all specifications. “a” is a test for fixed effects, “b” is the Hausman test, “c” is the Swami test for coefficient homogeneity, while “d” is a Hausman type test for coefficient homogeneity.

Parameter homogeneity is now overwhelmingly rejected

Implies any estimate but RCM will be biased upwards

RCM implies half-life of one year and two months.

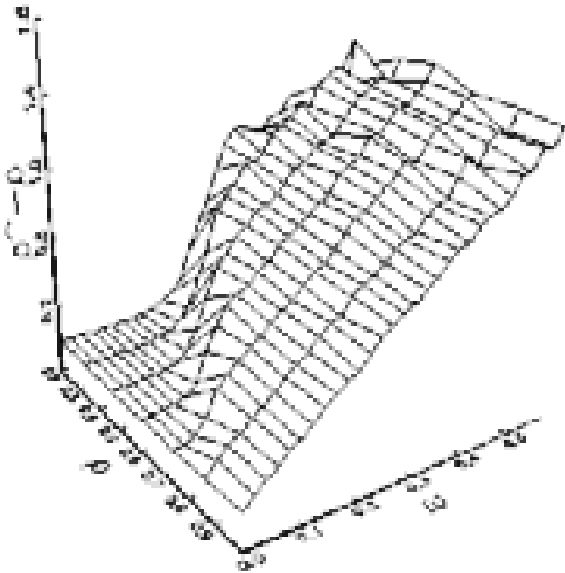
# How big is the bias? Monte Carlo I



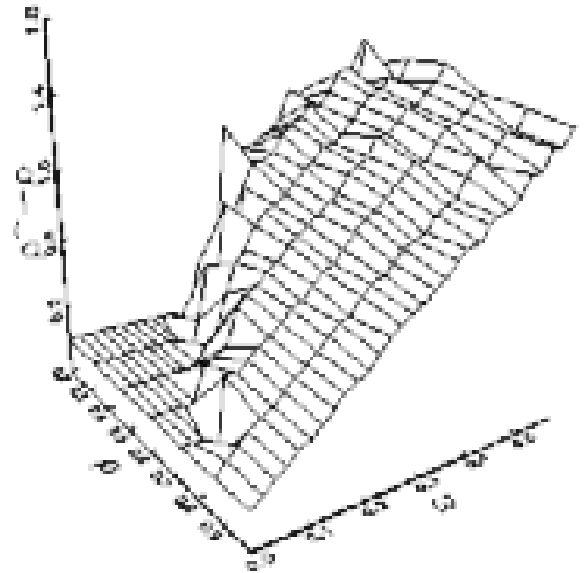
Sum of all autoregressive coefficients, simulated using DGP implied by RCM estimation.

# How big is the bias? Monte Carlo 2

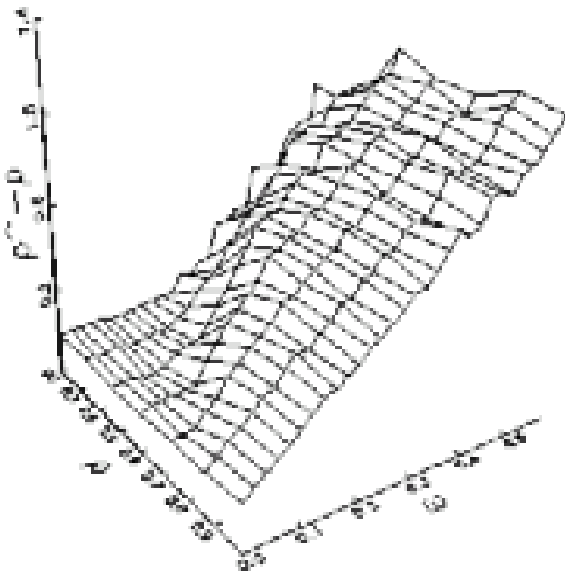
Fixed Effects



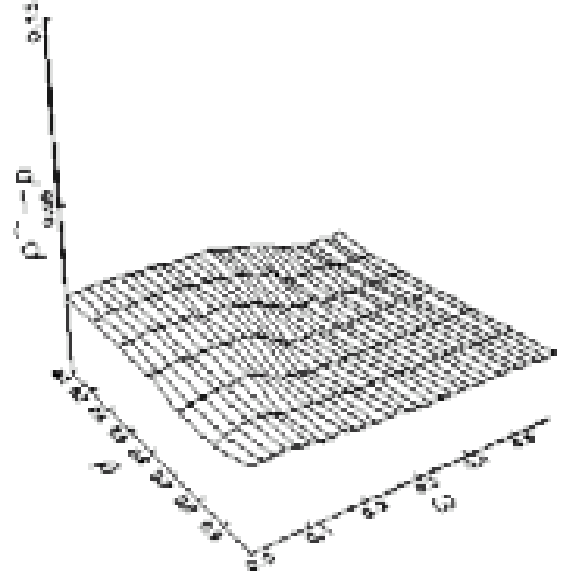
Anderson Hsiao



Aggregated Fixed Effects



Random Coefficients





## Does it matter? – Chari et al (2002)

Statistic	Variations on the Benchmark Economy <sup>a</sup>							
	Data <sup>b</sup>	Benchmark Economy	High Exports	Nonseparable Preferences	Real Shocks	Taylor Rule	Sticky Wages	Incomplete Markets
<i>Standard Deviations Relative to GDP<sup>c</sup></i>								
Price ratio	.71	3.00 (.75)	3.26 (.77)	.02 (.00)	2.98 (.74)	1.35 (.33)	2.11 (.59)	2.98 (.75)
Exchange rate								
Nominal	4.67	4.32 (.80)	4.27 (.79)	.07 (.01)	4.27 (.80)	4.66 (.66)	4.14 (.80)	4.22 (.78)
Real	4.36	4.27 (.72)	4.09 (.67)	.05 (.01)	4.26 (.71)	4.98 (.72)	4.35 (.83)	4.19 (.71)
<i>Autocorrelations</i>								
Price ratio	.87	.93 (.02)	.92 (.02)	.81 (.06)	.93 (.02)	.92 (.02)	.95 (.02)	.93 (.02)
Exchange rate								
Nominal	.86	.69 (.08)	.69 (.08)	.83 (.05)	.69 (.08)	.46 (.10)	.69 (.08)	.69 (.08)
Real	.83	.62 (.08)	.58 (.08)	.77 (.06)	.62 (.08)	.48 (.09)	.69 (.08)	.62 (.08)
<i>Cross-Correlations</i>								
Real and nominal exchange rates	.99	.76 (.06)	.70 (.07)	.98 (.00)	.76 (.06)	.96 (.01)	.88 (.04)	.75 (.06)

\* See notes at the end of the tables.

Their preferred estimates for the autocorrelation coefficient = 0.62 (0.08)

When we simulate our corrected (RCM) model, we get an autocorrelation coefficient equal to 0.636 (0.08), as opposed to 0.83 in the data...

Thus, no PPP puzzle...

# Robustness: Alternative Explanations

## 1. Errors in Variables:

Sectoral data is more noisy. This results in an attenuating bias in sectoral estimates of persistence, which translates in the aggregate.

Is that happening?

First we don't observe systematically lower estimates at the sectoral level.

Second, use Hausman test to investigate whether IV and OLS yield same sectoral estimates (as they should without Error in Variables).

(Instruments are lags)

Hausman rejects consistency of OLS in only 14 of the 222 country-sectors. Table 4 reports estimates when using GMM for these 14 cases.

**Table 4: Errors in Variable in the Disaggregate Data**

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$$q_{it}^* = \sum_{p=1}^P \rho_{i,p} q_{it-p}^* + \nu_{it}$$

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Model	$P$	$\sum_{j=1}^P \rho_j$	Half Life	Confidence Interval
RCM <sup>1</sup>	5	0.94476	13	4,21
RCM <sup>2</sup>	5	0.94642	14	4,23

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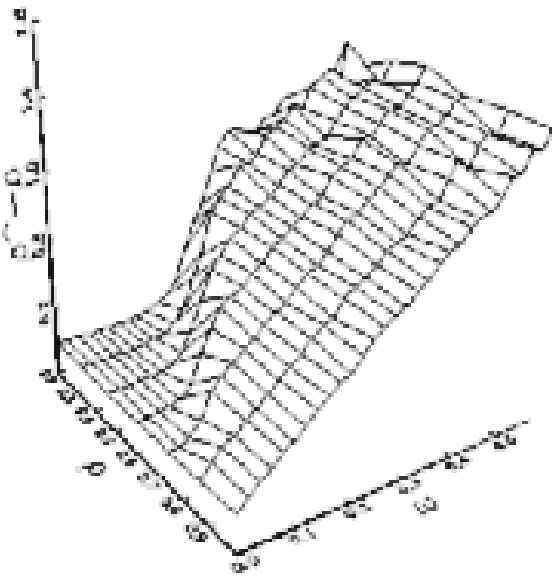
## 2. Attenuation Bias for Half-Life Estimates of Persistent Processes

In short samples, estimates for the half-life of very persistent processes are biased downward. Murray-Pappell applied the point to the RER, and revised its half-life estimates upwards (particularly in univariate context)

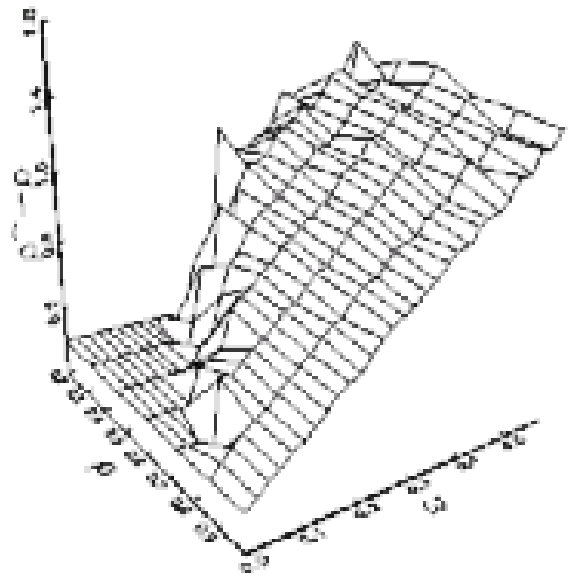
Could that be at play?

- (i) our sample is remarkably long (up to 264 monthly observations). Monte-Carlo simulations suggest it may be long enough (Rossi (2002))
- (ii) sectoral relative prices are much less persistent than aggregate RER (Cf Table 1)
- (iii) Monte-Carlo in Figure 2 suggest the attenuating bias is dominated by the aggregation bias.

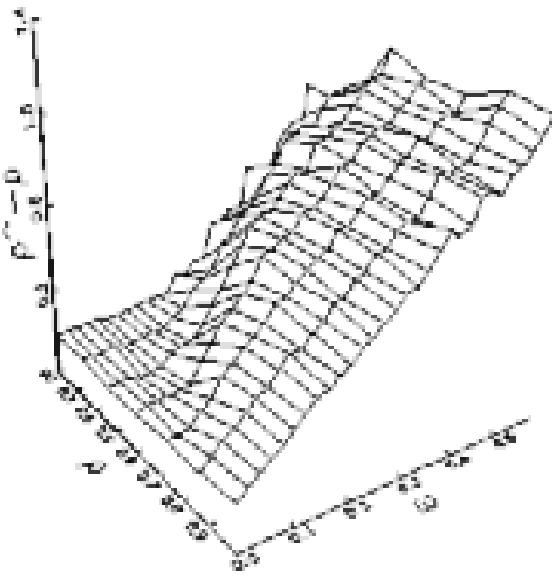
Fixed Effects



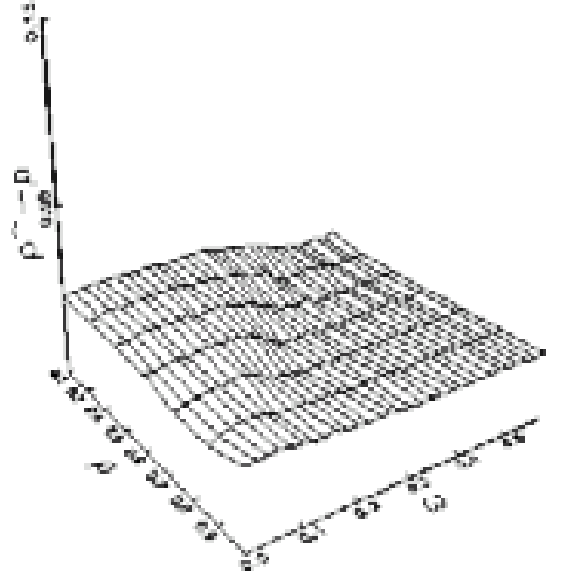
Anderson Hsiao



Aggregated Fixed Effects



Random Coefficients



## Traded vs. Non-Traded Goods

Engel (1999) showed the vast majority of the persistence and volatility in the RER can be attributed to traded goods prices, even at long horizons.

Surprising, since mean reversion should be expected to be faster amongst precisely these goods. Rules out explanations of the PPP puzzle based on tradeability.

Natural explanation for this anomaly is if heterogeneity is larger amongst traded goods, because then aggregation bias will be more acute there.

Split our data according to same split as in Engel to investigate this possibility.

## Conclusion

- Research on deviations from LOP directly imply cross-sectoral heterogeneity, empirically and theoretically.
- Despite this wealth of evidence, heterogeneity rarely if ever taken into account during estimation
- When accounted for, half-life of real exchange rate mean reversion fall to a little more than one year, with a confidence interval that breaks the “consensus view”, but from below...
- Corrected estimates are consistent with plausible nominal rigidities, thus, arguably, solving the PPP puzzle.
- Heterogeneity also offers answer to puzzling dynamics of traded goods prices.
- One should not expect standard estimates of the RER to mean revert quickly.