

*Harvard University*  
*Department of Economics*

**General Examination in Microeconomic Theory**

*Fall 2007*

1. You have **FOUR** hours.
2. Answer all questions

**PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.**

**PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.**  
**PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.**

1. Today, there are hundreds of thousands of homes facing foreclosure in the wake of the subprime lending crisis. One public policy path has been a moratorium on foreclosures, so that lenders cannot evict borrowers from their homes. The following questions ask about the consequences of policies such as that one, but it is perhaps wiser to model the policy not as a discrete ban on foreclosures but rather an increase in the cost of seizing a home. Do not worry about the time consistency of the change, but instead assume that the change happens ex ante and is known to be permanent.

- (a) Assume that there is only one type of borrower (ex ante at least) and one type of lender. Assume further that the number of homes is fixed. What will an increase in the cost of foreclosure do to (1) housing prices, (2) housing expenditure, (3) the welfare of borrowers and the (4) the welfare of lenders. Assume that everyone must own a home. (5 points)
- (b) Now consider a more general equilibrium setting where the housing supply is more elastic. How will the change in foreclosure rules change prices, expenditures, welfare and housing quantities. (5 points)
- (c) Now think about a world in which there are two types of prospective borrowers one group which is known to be very unlikely to have a negative income shock and one group which is much riskier. How will a change in the foreclosure rules affect their welfare and prices if they consume the same type of housing? How will a change affect their welfare if they consume different types of housing? (5 points)
- (d) What do you think that including renters would do to your analysis? (5 Points)

## Game Theory Question

Player 1 ("Seller") owns an object which is worth \$0 to him, and worth \$ $v$  to player 2 (the "Buyer"), with  $v > 0$ . Both players are expected utility maximizers, do not discount the future, and care only about monetary payoffs. [All parts of this question carry equal weight.]

1. Consider the following game: players simultaneously announce prices  $p_S$  (Seller's price),  $p_B$  (Buyer's price). If  $p_B \geq p_S$ , the buyer gets the object and pays  $p_S$  to the seller. If  $p_S > p_B$ , the game ends and the seller keeps the object.

- Find all strictly dominated strategies for each player.
- Find all weakly dominated strategies for each player.
- Find the set of Nash equilibrium outcomes.

2. Now modify the game as follows: in period  $t = 1$ , the seller offers a price  $p_1$ , which the buyer accepts or rejects. If the buyer accepts, he gets the object and pays  $p$  to the seller. If he rejects, then the seller makes another offer  $p_2$  in period 2, which the buyer again accepts or rejects. If the buyer accepts, sale occurs at price  $p_2$ ; if he rejects, the game ends and the seller keeps the object.

- Show that there is a unique subgame perfect Nash equilibrium (find it).
- Are there additional Nash equilibria? If yes, find one. If no, support your answer.

3. Again consider the game from part (2) above, but now assume there are two possible types for the Buyer: type H, with (high) valuation  $v = 2$ , and type L, with (low) valuation  $v = 1$ . The buyer is type H with probability  $x$ . Find a Perfect Bayesian equilibrium (PBE), through the following steps:

- Explain the following (correct) statement:

In any PBE: if agreement is not reached in period 1, then in period 2, a buyer of type L must accept any price  $p \leq 1$  with probability 1, and a buyer of type H must accept any price  $p \leq 2$  with probability 1. The seller offers either  $p = 1$  or  $p = 2$ , depending on his beliefs about the buyer.

- Now calculate the seller's optimal strategy(s) in period 2, if agreement was not reached in period 1, as a function of his beliefs about the buyer's type.

- Explain the following (correct) statement:

In any PBE: in period 1, the buyer of type L must accept any  $p \leq 1$  with probability 1, and reject any  $p > 1$  with probability 1. If  $x < \frac{1}{2}$ ,

the buyer of type  $H$  must accept  $p \leq 1$  w.p. 1, and reject  $p > 1$  w.p. 1.

- d. What happens (in a PBE) if  $x > \frac{1}{2}$ , the buyer is in fact type  $H$ , and the seller offers  $p > 1$  in period 1?

## 1. Possible failures of first welfare theorem

a. Suppose that there are two agents and two commodities. Both agents have differentiable, strictly increasing and strictly concave utility, but there is a missing market for commodity 2. Define Walrasian equilibrium and show in an Edgeworth box (or analytically if you prefer) that typically Walrasian equilibria are not Pareto-efficient here.

b. Suppose that there are two households with identical utility  $u^h(x) = \log(x_1) + \log(x_2)$ ,  $h = 1, 2$  and endowments  $e^1 = (1, 2)$  and  $e^2 = (2, 1)$  respectively. Compute a Walrasian equilibrium and show that it is Pareto-optimal. Now assume that agent 1 sets a price, agent 2 chooses optimal demand at this price and agent 1 supplies and demands the right amount so that markets clear. Compute this non-competitive equilibrium and show that it is not Pareto-optimal.

c. Suppose there is the following externality in the economy. Let  $x^1 = (x_1^1, x_2^1)$  be household 1's consumption, let  $x^2 = (x_1^2, x_2^2)$  be household 2's consumption. The two households have utility functions

$$u^1(x) = \log(x_1^1 + x_1^2) + \log(x_2^1)$$

and

$$u^2(x) = \log(x_1^2) + \log(x_2^2).$$

Suppose endowments are as in b), but agents behave competitively. Compute all Pareto-optimal allocations and compute one Walrasian equilibrium. Is it Pareto-optimal?

a) What is the Revelation Principal for Dominant Strategy Mechanisms? Define any technical terms you use.

b) State the Gibbard-Satterthwaite Theorem.

c) Consider the following situation, sometimes called "King Solomon's Dilemma". There is a baby and two possible mothers for the baby, who are players 1 and 2. One and only one of the players is the true mother. The identity of the true mother is common knowledge to them but is unknown to King Solomon.

There are three outcomes that King Solomon (the mechanism designer) can impose as a result of the mothers' play: give the baby to player 1 (call that outcome  $x_1$ ), give the baby to player 2 (call that outcome  $x_2$ ), or cut the baby in half (call that outcome  $x_3$ ). If the true mother is player 1, player 1 preferences are  $x_1 \succ x_2 \succ x_3$ , and player 2's preferences are  $x_2 \succ x_3 \succ x_1$ . Likewise, if the true mother is player 2, player 2's preferences are  $x_2 \succ x_1 \succ x_3$ , and player 1's preferences are  $x_1 \succ x_3 \succ x_2$ . Does the Gibbard-Satterthwaite Theorem apply to King Solomon's Dilemma? Be specific about which of the conditions required by the theorem are or are not satisfied.

d) Would it help King Solomon if there were a transferable resource such as money that enters quasi-linearly into the preferences of both mothers, modifying the preferences above (which would be their preferences when monetary payments were zero).

e) Assuming that such a transferable resource exists, what mechanism would you recommend that King Solomon use to implement an outcome function with the property " $x_1$  if 1 is the true mother and  $x_2$  if 2 is the true mother"?