

# A Biased View of PPP

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# Road Map

- Current Debate
- Introduction to the PPP puzzle
- Theory: small sample and heterogeneity biases
- Empirical evidence
- Monte-Carlo analysis
- Conclusion

# Current debate

- Imbs et al (2004): sectoral heterogeneity bias can explain the PPP puzzle
  - Use monthly disaggregated data at the sectoral level from Eurostat, 1981-1995.
  - Theoretical conditions for heterogeneity bias are present in the data.
  - Apply Pesaran (2002) estimator to deal with sectoral data and find that PPP puzzle disappears. Half-life is around 1 year.

# Current debate

- Chen, Engel (2004): sectoral heterogeneity bias is irrelevant
  - Use similar data as Imbs et al (2004).
  - Use Monte Carlo simulations to show that in small samples heterogeneity bias is not enough to solve the PPP puzzle.
  - Their simulations focus on OLS time series estimates (not panel data).
  - Their sample sizes in the simulations run up only to around 40 years (500 monthly data points).
  - Do not provide a reason why Imbs et al (2004) obtain such low estimates.

# Current debate

- Choi, Mark, Sul (2003, 2004): country heterogeneity bias is irrelevant
  - Use annual data for 21 OECD countries.
  - Use the Recursive Demeaning Estimator to alleviate the small sample bias
  - Do not explore sectoral heterogeneity but find that country heterogeneity is irrelevant.
  - Their sample sizes in the simulations run up only to around 200 data points.
  - Do not provide a reason why Imbs et al (2004) obtain such low estimates.

# Our Methodology

- Use disaggregated data as in Imbs et al (2004)
- Explore both time series and panel estimations
- Use OLS, SURE and CCE estimators
- Use improved Monte Carlo engine
  - non-diagonal covariance structure across sectors
  - calibrated for different data frequencies (monthly and annual)
  - short, medium and large samples (up to 10,000 points)
  - general structure of heterogeneity
  - Information-neutral initial conditions

# Our results

We explore the interaction between small-sample and heterogeneity biases that plague estimates of half-lives of PPP deviations:

- in small samples heterogeneity bias can disappear completely.
- heterogeneity bias does not resolve the PPP puzzle.
- for longer series heterogeneity bias dominates small sample bias and results in upward-biased estimates.
- Imbs et al (2004) find low half-lives because they use Pesaran (2002) estimator which is not applicable to this problem.

Part I  
Introduction to the PPP Puzzle

# Part I – The puzzle ...

At the heart of the debate is a "disarmingly simple proposition" (Rogoff (1996)) that a basket of goods in country B, when expressed in currency units of country A, should cost the same as the basket in country A. In other words:

$$PPP: Q = \frac{P^A}{P^B E} = 1$$

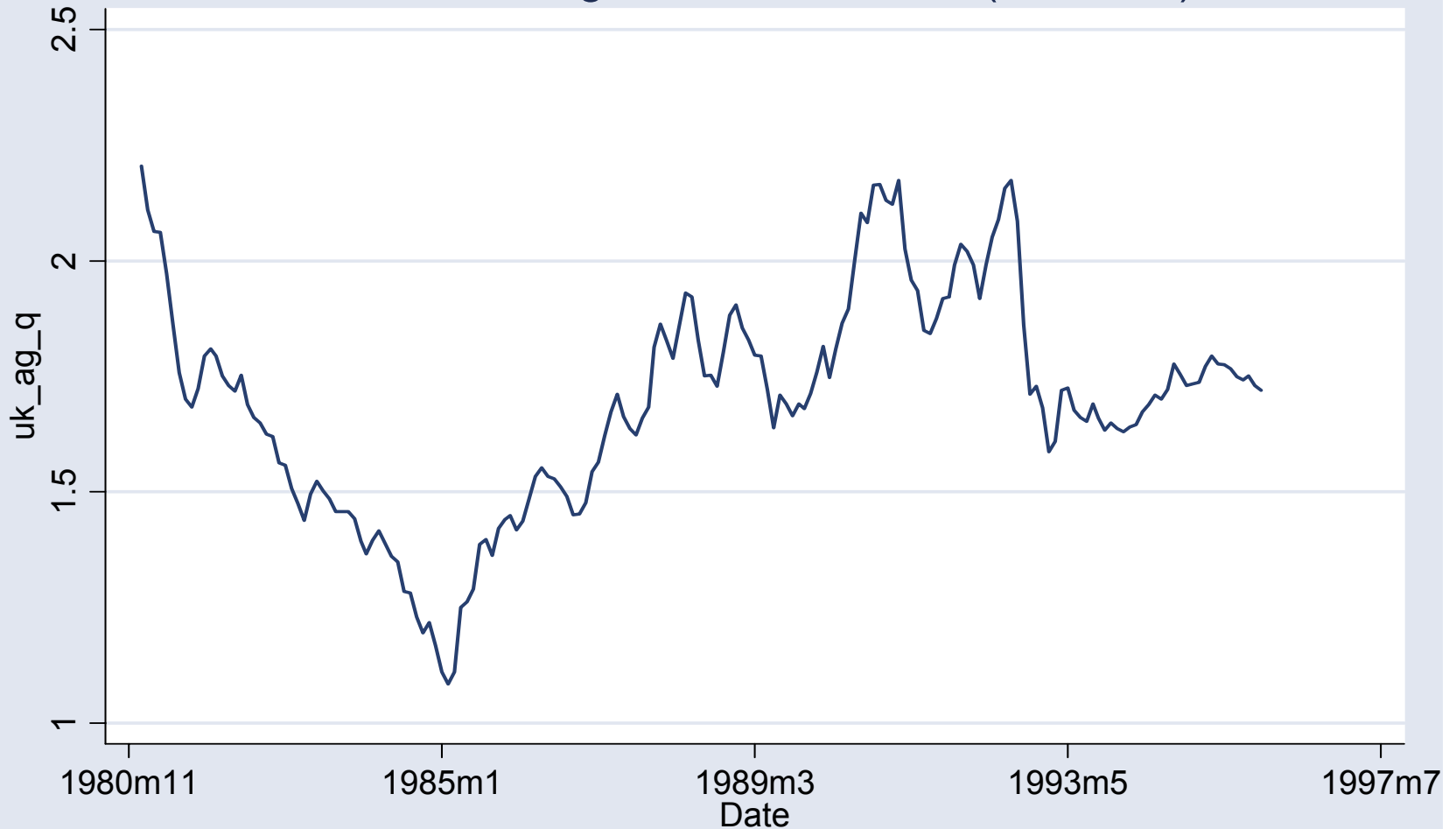
# Part I – The puzzle ...

However, it is evident from the data that deviations from PPP are large and persistent.

The “consensus view” (Rogoff (1996)) is that the average half-life of these deviations is 3-5 years, a fact difficult to square with any model with nominal rigidities.

# Theory vs. Reality

Real Exchange Rate movement (U.K/U.S.)



# Part I – Biases in estimation

Estimations of half-lives are particularly affected by two different biases:

1. Small Sample Bias (Orcutt (1948), Kendall (1954), ...)
2. Heterogeneity Bias (Pesaran and Smith (1995), Imbs et al (2004))

# Part I – Biases in estimation

Imbs et al (2004) actually claim that the effect of heterogeneity is large enough to solve the puzzle.

They find that in the data, heterogeneity creates an upward biased estimate of the half-lives of deviations from PPP.

Their bias-corrected estimates are close to one year, in contrast to the “consensus view” of 3-5 years.

## Part II

# Theory: Small Sample and Heterogeneity Biases

# Part II – Heterogeneity bias

- Heterogeneity bias in time-series:

*Sectoral real exchange rate:*

$$\begin{cases} q_{it} = c_i + \rho_i q_{it-1} + \varepsilon_{it} \\ \rho_i = \rho^{true} + \eta_i^\rho, c_i = c^{true} + \eta_i^c \end{cases}$$

*Estimated model:*

$$\begin{cases} q_t = c + \rho q_{t-1} + \varepsilon_t \\ c = \sum_i \omega_i c_i, \varepsilon_t = \sum_i \omega_i \varepsilon_{it} + \sum_i \omega_i \eta_i^\rho q_{it-1} \end{cases}$$

$$\text{Bias: } \tilde{\rho} = \rho + \Delta, \Delta = \sum_i (\rho_i - \rho) \alpha_i$$

$$\alpha_i = \left( \frac{\omega_i^2 \sigma_i^2}{1 - \rho_i^2} + \sum_{j \neq i} \frac{\omega_i \omega_j \sigma_{ij}}{1 - \rho_i \rho_j} \right) / \sum_i \left( \frac{\omega_i^2 \sigma_i^2}{1 - \rho_i^2} + \sum_{j \neq i} \frac{\omega_i \omega_j \sigma_{ij}}{1 - \rho_i \rho_j} \right)$$

# Part II – Heterogeneity bias

- Key idea: the estimated  $\rho$  from aggregate series is a biased estimator of the average  $\rho$  in the economy (Pesaran, Smith (1995)).  
Therefore, we cannot interpret this number as the average speed of adjustment (which is the PPP puzzle)
- It does NOT mean that the estimates of  $\rho$  for the real exchange rate are wrong or biased.

# Part II – Heterogeneity bias

- A simple illustration:

Assume that  $\sigma_{ij} = 0$ ,  $\sigma_i^2 = \sigma^2$ ,  $\omega_i = \frac{1}{N}$

$$\alpha_i = \frac{1/(1-\rho_i^2)}{\sum_j 1/(1-\rho_j^2)}, \text{ positive and increasing in } \rho_i$$

$$\text{Therefore } \Delta = \sum_i (\rho_i - \rho)\alpha_i > 0$$

# Part II – Small sample bias

- Small sample bias in OLS estimates:

*Let  $\vec{q}_T$  be the real exchange rate vector and assume that*

$$\vec{q}_T = \rho \vec{Q}_{T-1} + \varepsilon_T$$

*Then*

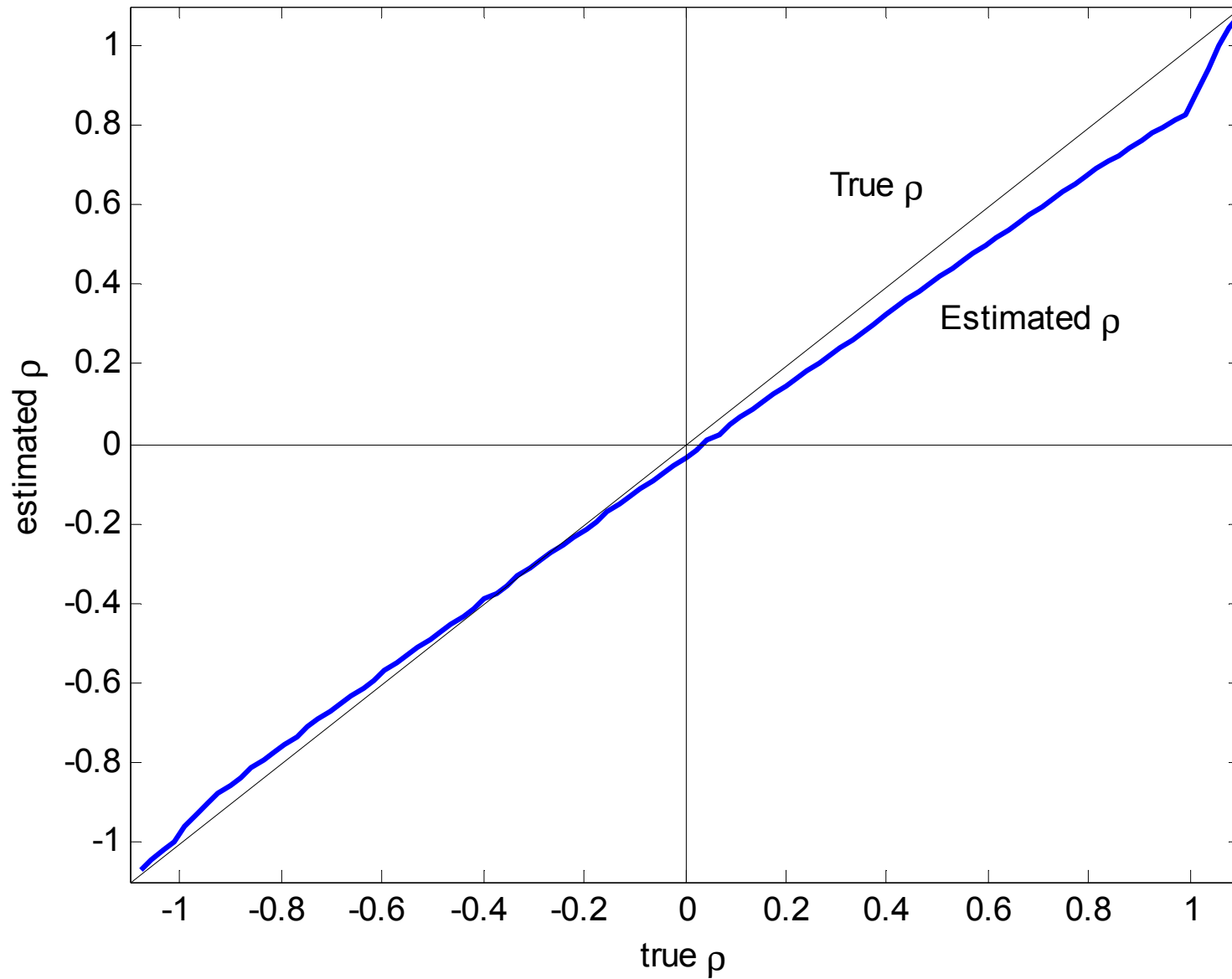
$$\hat{\rho} = (\vec{Q}_{T-1}' \vec{Q}_{T-1})^{-1} \vec{Q}_{T-1}' \vec{q}_T$$

$$E[\hat{\rho}] = \rho + E[(\vec{Q}_{T-1}' \vec{Q}_{T-1})^{-1} \vec{Q}_{T-1}' \varepsilon_T]$$

*and*

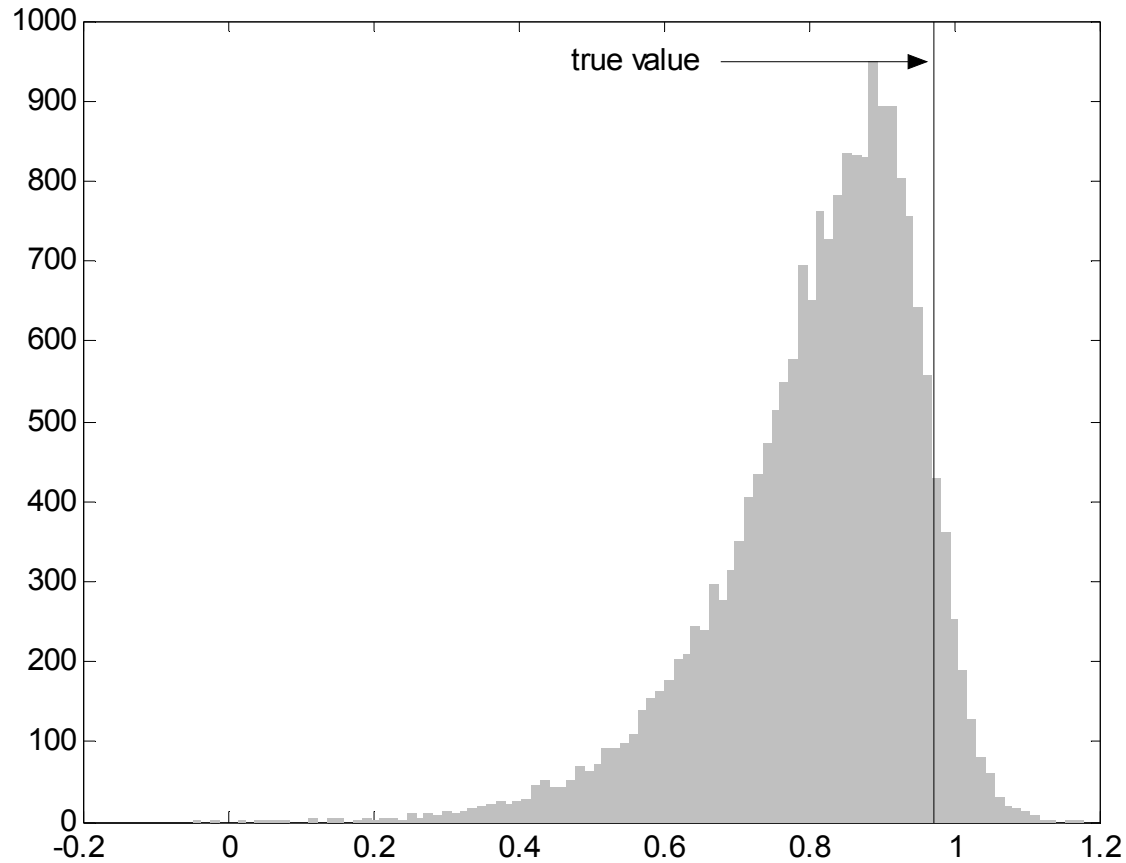
$$E[(\vec{Q}_{T-1}' \vec{Q}_{T-1})^{-1} \vec{Q}_{T-1}' \varepsilon_T] \xrightarrow{p} E[(\vec{Q}_{T-1}' \vec{Q}_{T-1})^{-1}] E[\vec{Q}_{T-1}' \varepsilon_T] = 0$$

Estimation of  $\rho$  (OLS) in a small sample ( $T=20$ ,  $\rho=.97$ )



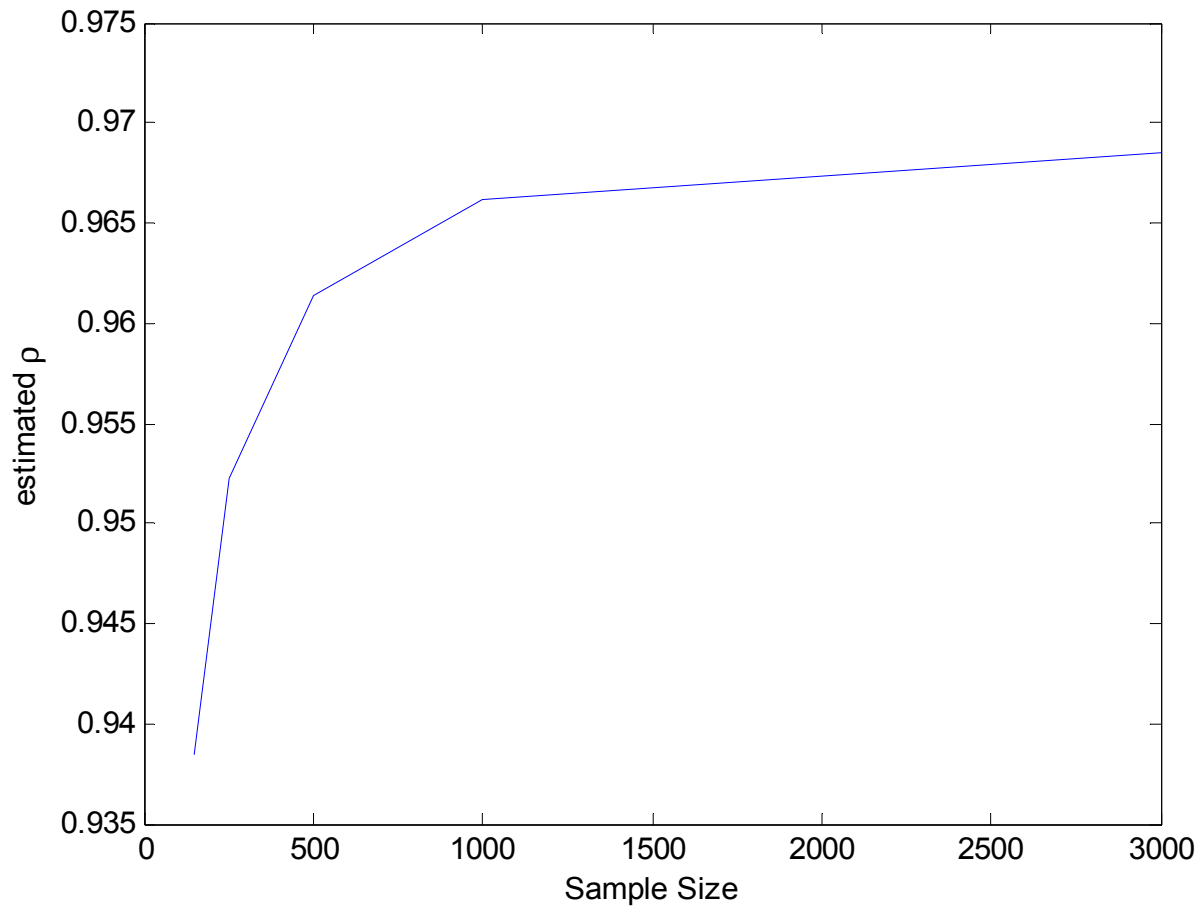
# Histogram of OLS estimator

$$\rho_{\text{true}} = 0.97, T = 20$$



# Small sample bias as a function of sample size

$$\rho_{\text{true}}=0.97$$



# Part II – Small sample bias

- There exist several correction methods: Kilian(1998) bootstrap-after-bootstrap, Andrews (1993) exactly median-unbiased estimator, etc.
- We use Andrews (1993) median-unbiased estimator because Kilian (1998) method performs poorly for processes with very high persistence.

# Part II – Interaction

- Note that small-sample bias, by definition, appears in small samples.
- In contrast, the analytical results for heterogeneity bias are valid ONLY asymptotically.
- We do not have analytical solutions for their interaction in finite samples. In Part IV we use Monte Carlo simulations to explore this interaction.

# Part III

## Empirical Evidence

# Part III – Empirical results

## Dataset:

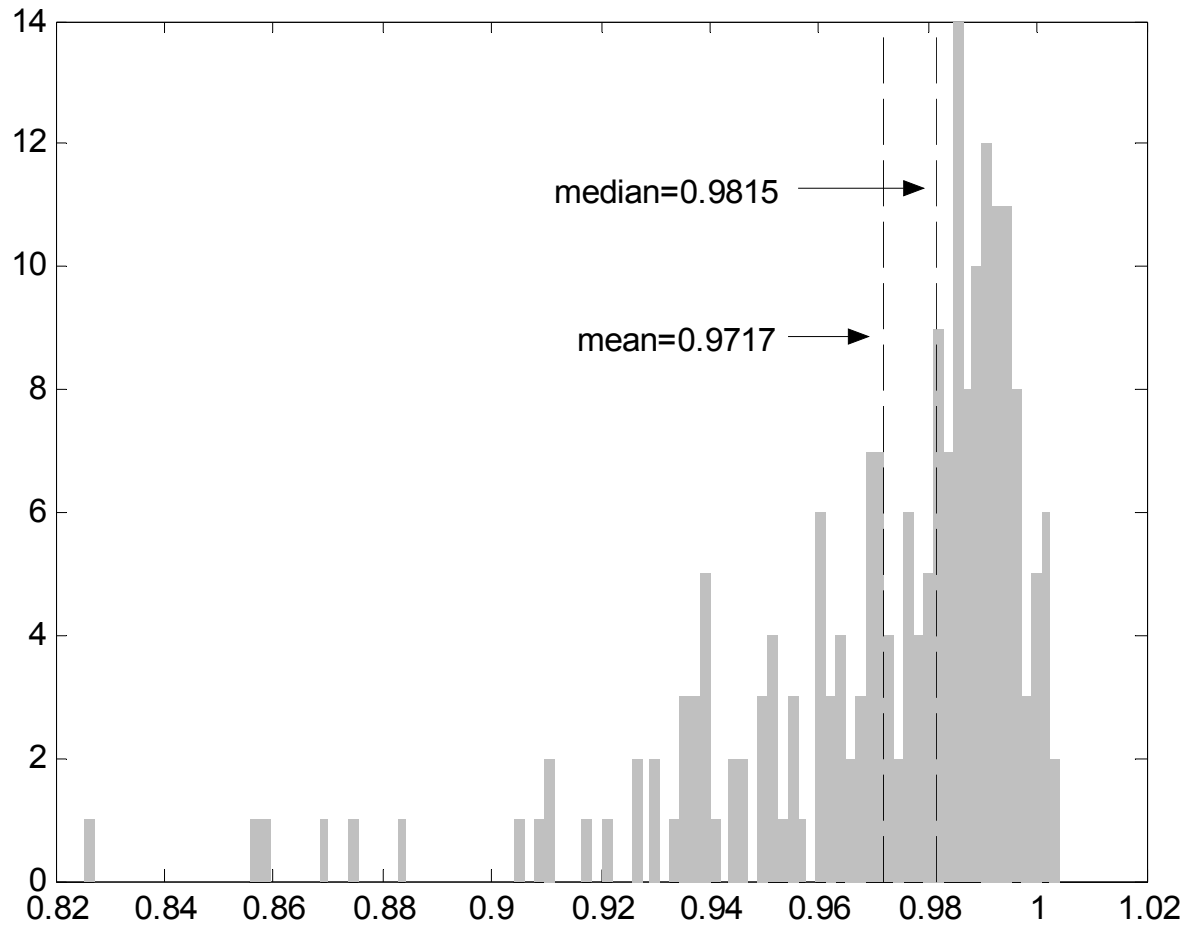
- Eurostat data for Belgium, Germany, Denmark, Finland, France, Greece, Italy, Netherlands, Portugal, Spain, U.K.
- 19 different sectors including “meat,” “dairy,” “clothes,” etc.
- Time range from 1981-1995, which gives them a maximum length of 180 datapoints

# Part III – Empirical results

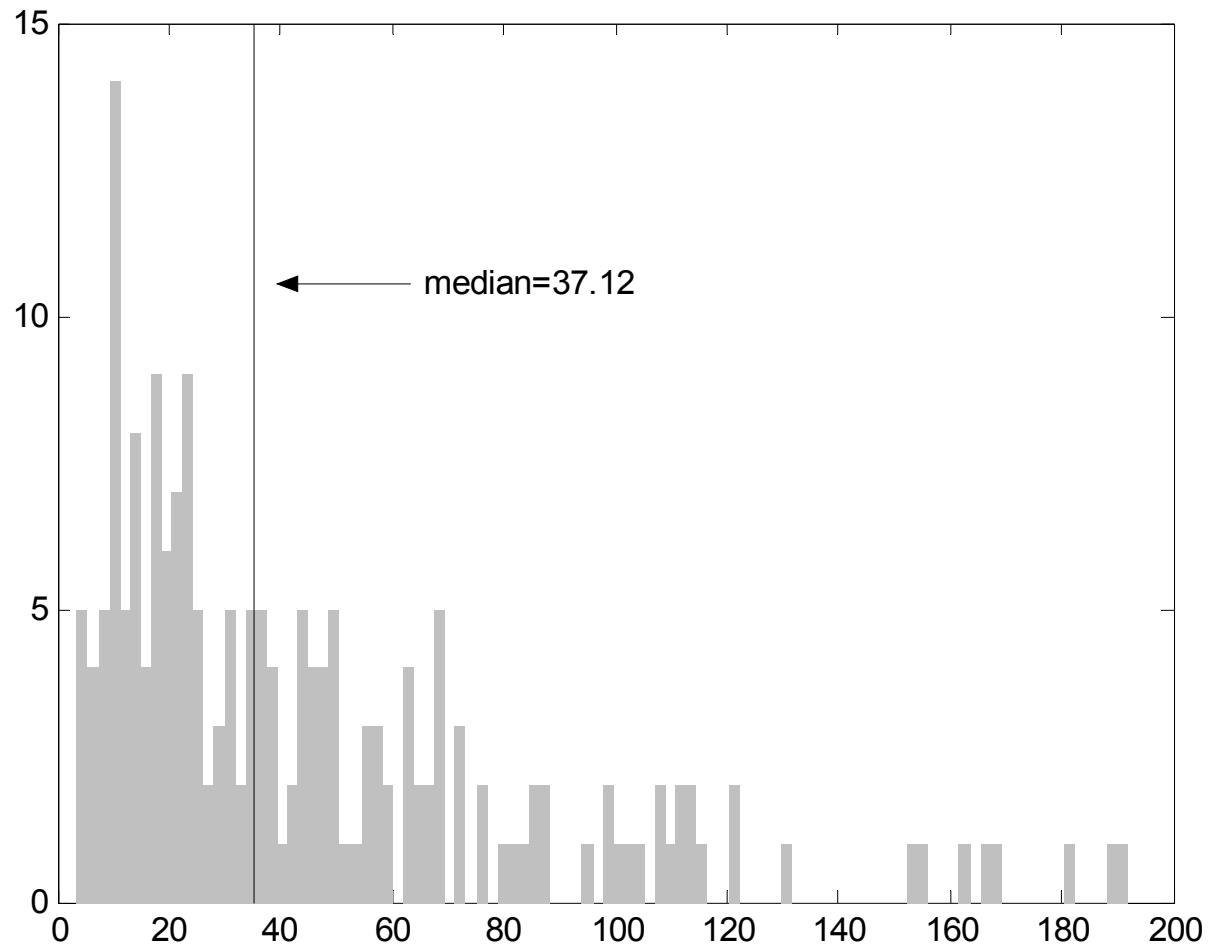
Main findings:

- There is significant sectoral heterogeneity
- Theoretical conditions for heterogeneity bias are satisfied in the dataset
- Individual sectors exhibit long half-lives (even before correcting for small-sample bias)

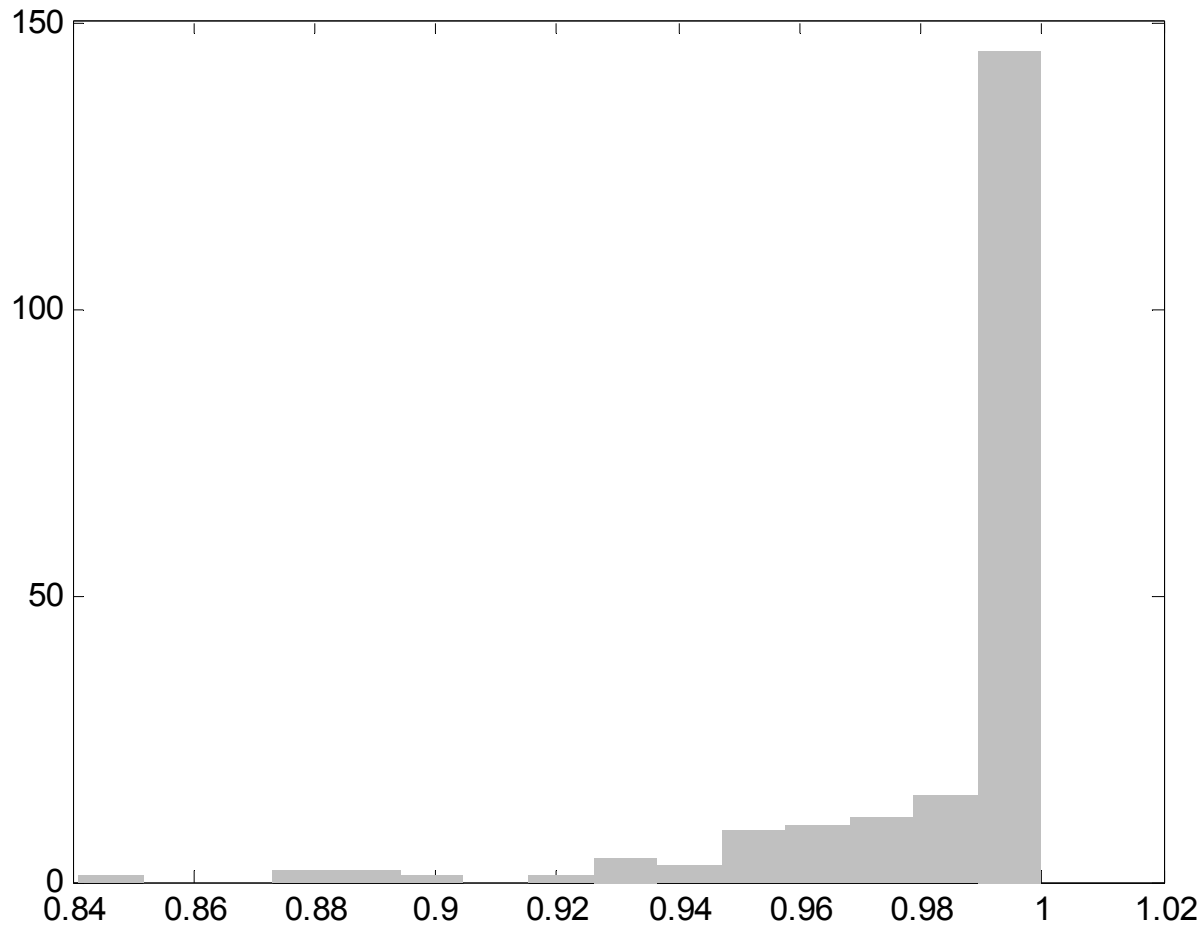
# Distribution of $\rho$ for different sectors and countries (without correcting for small sample bias)



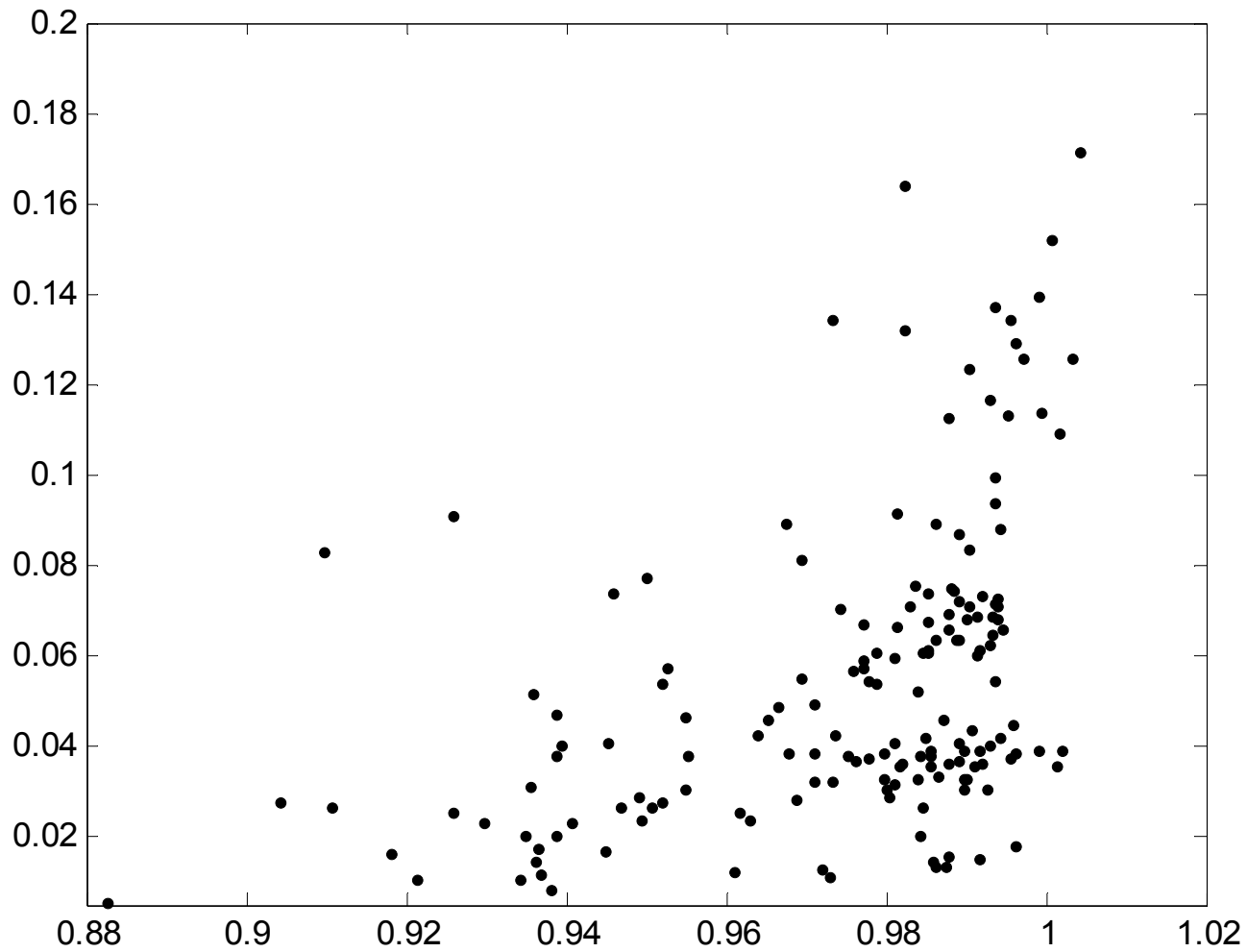
# Distribution of half-lives for different sectors and countries (without correcting for small sample bias)



# Distribution of $\rho$ for different sectors and countries (correcting for small sample bias)



Scatter plot:  $\alpha$  (weight) vs.  $\rho$



# Part III – Empirical results

- Empirical results cast doubt on the ability of heterogeneity bias to resolve the PPP puzzle.
- However, this does not imply that the effect is irrelevant.

# Part IV

## Monte Carlo Analysis

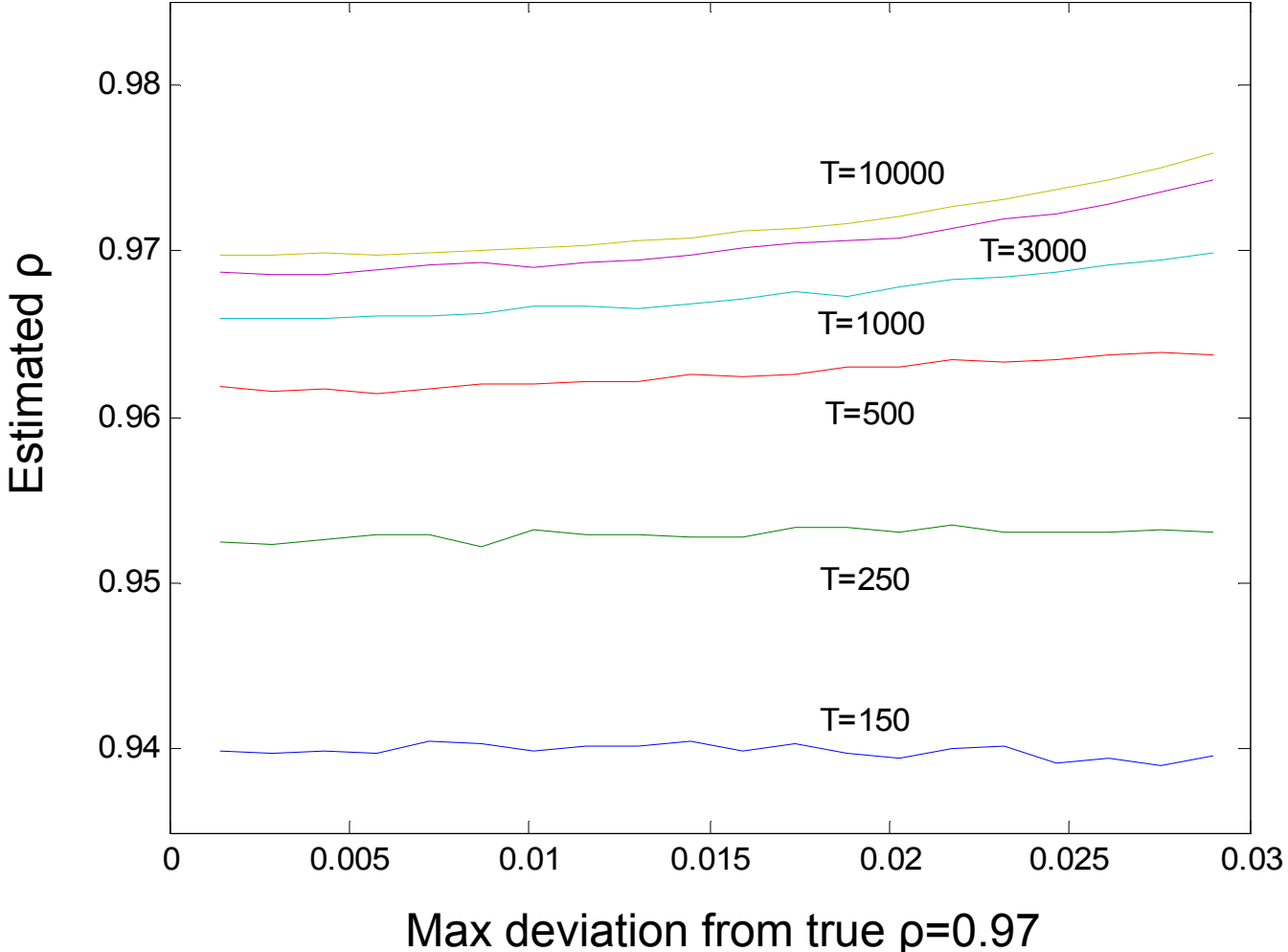
# Part IV – Plan

- Interaction of the two biases in time-series data: annual and monthly frequencies.
- Interaction of the two biases in disaggregated data: CCE/MG and SURE/MG estimators.

# Part IV – Time Series Data

- Draw individual  $\rho_i$  from a specified distribution and simulate sectoral real exchange rate movement using a general cross-sectoral correlation matrix.
- We initialize using the unconditional distribution of the process.
- Finally, we estimate the half-life using OLS on the aggregate real exchange rate.

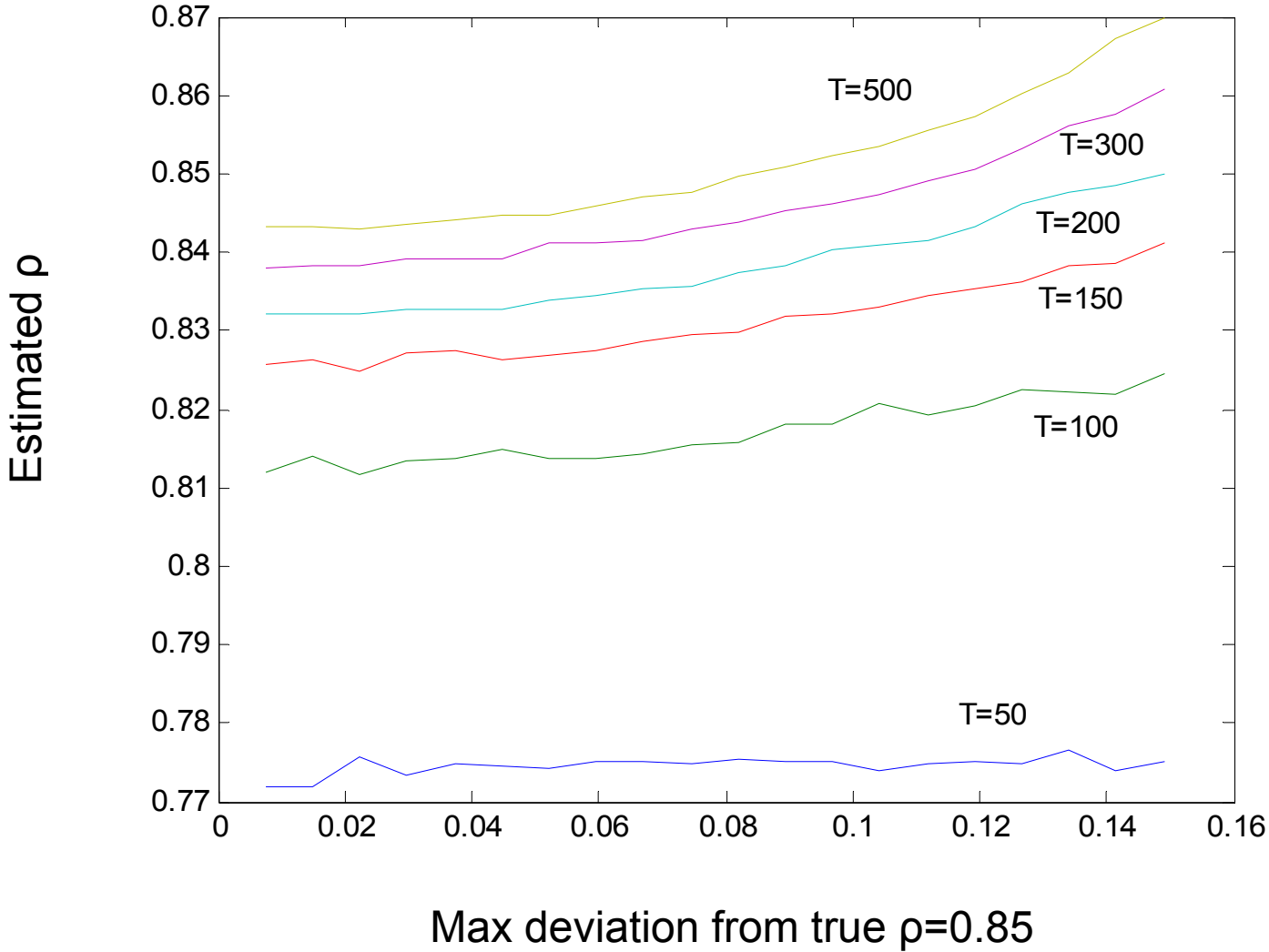
# Estimation of $\rho$ under heterogeneity and small sample biases (calibrated for monthly data)



# Part IV – OLS monthly

- For small samples (less than 40 years worth of data) heterogeneity bias disappears.
- The data used in Imbs et al (2004) is less than 20 years. Therefore, heterogeneity bias cannot explain the PPP puzzle.
- However, for longer series heterogeneity bias becomes relevant and may dominate the small sample bias.

# Estimation of $\rho$ under heterogeneity and small sample biases (calibrated for yearly data)



# Part IV – OLS annually

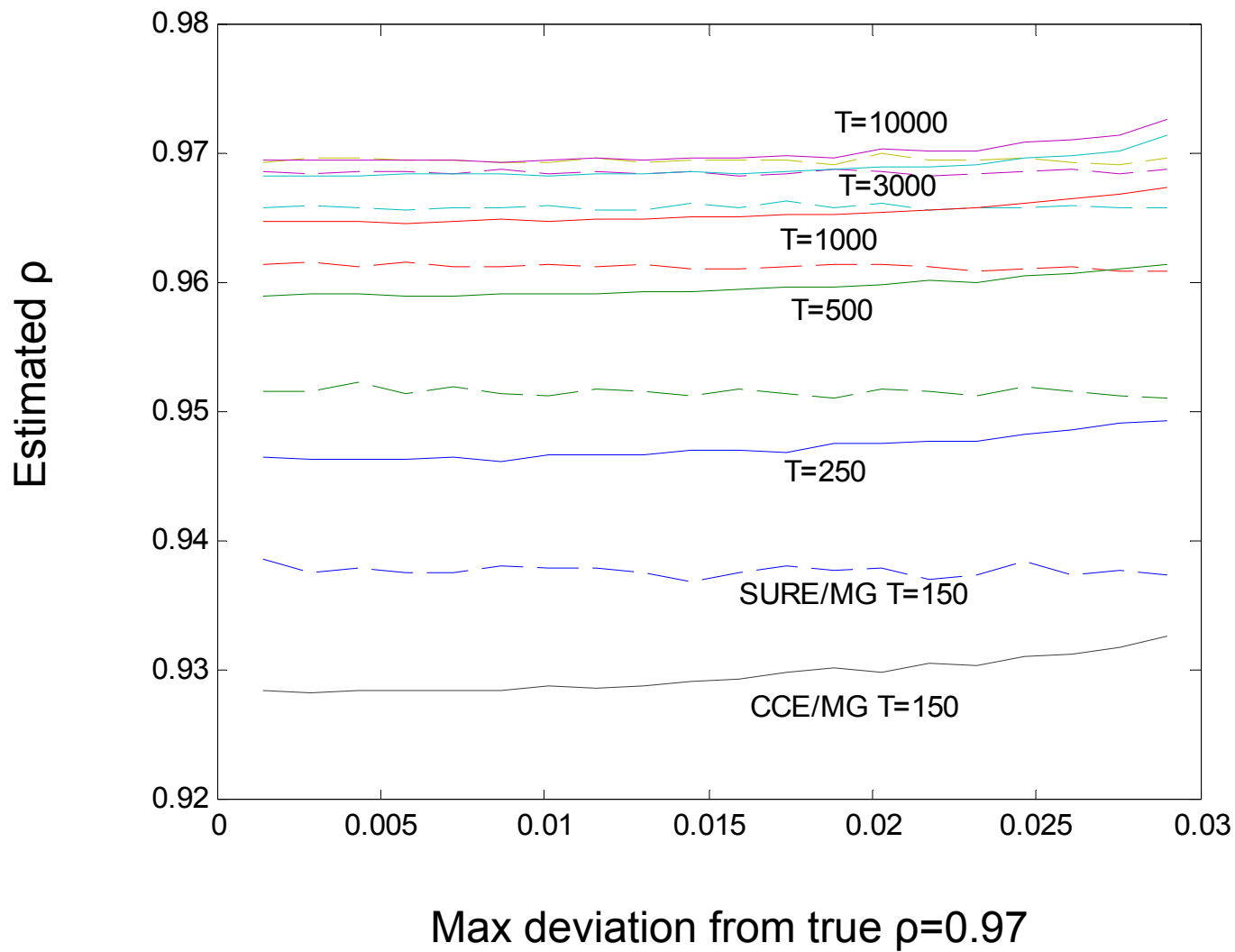
- Again, heterogeneity bias not important for 50 years or less worth of data.
- However, for series of 100 years or more heterogeneity will be present.
- Any small-sample bias adjustment will thus result in “overcorrection” implying upward-biased estimates of half-lives. Several studies have used 100 or more years of data (e.g. Lothian, Taylor (1996))

# Part IV – Panel Data

- Two estimators: CCE/MG (Pesaran (2002)) and SURE/MG.
- Imbs et al. (2004) use CCE/MG to show that heterogeneity bias solves the PPP puzzle.
- CCE/MG designed to deal with cross-sectional correlation in panel data. In particular, it's the OLS estimation of the following model:

$$\vec{q}_{iT} = \rho \vec{Q}_{iT-1} + \beta_1 \vec{q}_{T-1} + \beta_2 \vec{q}_T + \varepsilon_{iT}$$

# CCE and SURE under heterogeneity and small sample biases (calibrated for monthly data)



# Part IV – Panel Data

- Only CCE/MG is affected by the heterogeneity bias.
- Moreover, CCE/MG is affected by the small-sample bias much more than the SURE/MG estimator.
- This could explain why Imbs et al. (2004), using this method, obtain such low estimates of half-lives.
- In fact, the theoretical conditions for validity of CCE as in Pesaran (2002) are violated here.

# Part V

## Conclusion

# Part V – PPP puzzle remains

- Empirical evidence shows that sectoral heterogeneity is significant.
- Conditions for the heterogeneity bias to exist are satisfied.
- However, a majority of sectors have half-lives that are very long.
- Moreover, heterogeneity disappears in small samples and is hence unable to resolve the PPP puzzle.

# Part V – Heterogeneity matters

- In longer samples, heterogeneity bias dominates the small sample bias.
- Not taking its effects into account leads to “overcorrection” when adjusting for small-sample bias.
- CCE estimator seems to have lower variance but it is affected to a greater degree by both heterogeneity and small sample biases.