Econometrics Review

Honors Exam Preparation

April 10th 2008

Disclaimer

- NOT an exhaustive list of material
  - Ec1126 – Focus on theory
  - Ec1123 – Focus on application
- Advised to review first 8 topics for 1123
- No direction given for 1126
- Refer to lecture notes and text book for formulas and “formal” definitions

Suggested Study

1. Read the book /lecture outlines
2. Review homework solutions
3. Review previous exams
4. Practice book exercises

Introduction

- Causality vs. Forecasting (Predicting)

- Data types
  - Cross Sectional – “Snap shot”
  - Panel – Multiple periods and entities
  - Time series – Multiple periods

Sampling

- Population data hard to analyze
  - Difficult or impossible to collect
- Collect sample from population
  - Easier to work with
  - Representative of population?

Sampling Distribution

- Population joint distribution F

- Sample for i=(1, …, n)

  $(Y_{i1}, \ldots, Y_{im}, Z_{i1}, \ldots, Z_{iJ})$

- Random Sampling
  - Independent
  - Identically Distributed

Sampling Distribution

- It follows that sample is representative of population
- Law of large numbers
  - Sample mean converges in probability to population mean as n approaches infiniti
- Central limit theory
  - Sample mean is asymptotically normally distributed if random variable has a finite variance

Linear Regression

- Model: $Y_i = \beta_0 + \beta_1 X_i + u_i$

- Terminology
  - Independent variable (Regressor)
  - Dependent variable
  - Intercept
  - Coefficient(s)
  - Error term

OLS

- Ordinary Least Squares (OLS)
  - min $E(Y_i - \beta_0 - \beta_1 X_i)^2$ with choice variables β_0, β_1
- Assumption I:
  - $(Y_i - \beta_0 - \beta_1 X_i)$ orthogonal to X_i
- Assumption II:
  - $(X_i Y_i)$ are iid from joint distribution
- Assumption III:
  - Large outliers unlikely

OLS

- β_1 = Cov(X,Y)/Var(X)
- By LLN S_{xy} and S_x are consistent estimators of population covariances above
- Coefficient estimates are consistent estimators of population coefficients
  - $E(Y| 1, X) = \beta_0 + \beta_1 X_i$

OLS

- By LLN and CLT…
- Coefficient estimates are both consistent estimators of population coefficients and are asymptotically normally distributed

Hypothesis Tests & Confidence Intervals

- Hypotheses
  - H_o = Null
  - H_a = Alternative (False Null)

- Two-sided vs. One-sided H_a

- Confidence Intervals

Standard Errors

- Homoskedasticity vs. Heteroskedasticity
  - Estimators unbiased and asymptotically normal
  - Heteroskedastic → variance of $u|X$ not constant
  - Standard errors need to be corrected for heteroskedasticity
- When is OLS BLUE?
  - I-III + Homoskedasticity

Omitted Variables

- Does one regressor explain all of the variation in Y?
- Why does it matter? We're looking at relationship between X_1 and Y? Why do we care about X_2?
- Omitted Variable Bias
  - $Corr(X_1, X_2) \neq 0$
  - $Corr(Y, X_2) \neq 0$
  - Sign of Bias = Sign of $Corr(X_1, X_2)$ * Sign of $Corr(X_2, Y)$
    - Positive = Coefficient was previously biased upward
    - Negative = Coefficient was previously biased downward

Multiple Regression Model

- Model: $Y_i = \beta_0 + \beta_1 X_{1i} + \ldots + \beta_k X_{ki} + u_i$
- Calculating OVB
  - $E(X_2 \mid 1, X_1) = \pi_0 + \pi_1 X_{1i}$
  - $E(Y \mid 1, X_1) = \beta_0 + \beta_1 E(X_{1i} \mid 1, X_1) + \beta_2 E(X_{2i} \mid 1, X_1)$
  - $E(Y \mid 1, X_1) = \beta_0 + \beta_1 X_{1i} + \beta_2 (\pi_0 + \pi_1 X_{1i})$
  - $E(Y \mid 1, X_1) = (\beta_0 + \beta_2 \pi_0) + (\beta_1 + \beta_2 \pi_1) X_{1i}$

Multiple Regressors

- Assumptions I-III
- Homoskedasticity vs. Heteroskedasticity
- Perfect multicollinearity
  - Assumption IV of OLS

Hypothesis tests & Confidence Intervals

- Confidence Interval calculation for single coefficient no different in a multiple regressor model

- Joint Hypothesis
  - F-test
  - Restrictions (q)

- Test Joint Hypotheses
  - H_o: $ß_1=ß_2=0$
  - H_a: $ß_1 \neq 0$ <u>and/or</u> $ß_2 \neq 0$
  - $q=2$

Nonlinear models

- Polynomial Model
  - $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X^2_{1i} + \ldots + u_i$
- Logarithmic Model
  - Linear-log $Y_i = \beta_0 + \beta_1 \ln(X_{1i}) + u_i$
  - Log-linear $\ln(Y_i) = \beta_0 + \beta_1 X_{1i} + u_i$
  - Log-log $\ln(Y_i) = \beta_0 + \beta_1 \ln(X_{1i}) + u_i$
- Interaction Model
  - $Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{1i} D_i$

Nonlinear models

- Effect of a change in X depends on more than just β_1

- Test Non-linearity
  - H_o: $\beta_2 = \ldots \beta_k = 0$
  - H_a: $\beta_2 \neq \ldots$ and/or $\ldots \beta_k \neq 0$

Logarithmic Interpretation

- Linear-log
  - $.01 * \beta_1$
- Log-linear
  - $100 * \beta_1$ %
- Log-log
  - β_1 %

Interaction Models

- Dummy variables (Binary variables)
- Types & Interpretation
  - Binary * Binary
  - Binary * Continuous
    - Different Intercept
    - Different Intercept and Slope
    - Same Intercept and Different Slope

Ec1123

STOP!

OVB Revisited

- Model: $Y_i = \beta_0 + \beta_1 X_{1i} + \ldots + \beta_k X_{ki} + u_i$
- Some variables are unobservable
- Panel data provides a way to eliminate the OVB caused by certain kinds of unobservables

Panel Data

- Structural Regression Function

  $E(Y_t \mid 1, Z_{t,..,} Z_T) = \theta_0 + \theta_1 Z_t + \ldots + \theta_2 W + \theta_3 A$

- Where:
  - Z_t is predictor (varies by time)
  - W and A are time-invariant variables
  - Assume A is not observed so there is no sample counterpart

Panel Data

- Develop linear predictors that have sample counter parts
  - $X' = (Z_1 \ldots Z_T \; 1 \; W \; X_{T+2} \ldots X_K)$
- Linear predictor of A
  - $E(A \mid X) = \gamma_1 X_1 + \ldots + \gamma_K X_K$
- Linear predictor of Y_t given X
  - $E(Y_t \mid X) = \theta_0 + \theta_1 Z_t + \theta_2 W + \theta_3 E(A \mid X)$

System of Linear Predictors

- For T=2
  - $E(Y_1 \mid X) = (\theta_1 + \theta_3\gamma_1)Z_1 + \theta_3\gamma_2 Z_2 + R$
  - $E(Y_2 \mid X) = \theta_3\gamma_1 Z_1 + (\theta_1 + \theta_3\gamma_2)Z_2 + R$
  - R is a placeholder
- Exploit the common coefficients
  - $(\theta_1 + \theta_3\gamma_1) - \theta_3\gamma_1 = \theta_1$
- However there remain unidentified coeffs.

Autoregression

- Structural Model:
  - $E(Y_t \mid 1, Y_1, \ldots, Y_{t-1}, A) = \lambda_t + \theta Y_{t-1} + A$
- Restrictions
  - Exclusion: Assume only Y_{t-1} matters
  - Simple functional form: Partial effects constant

Setup

- Start with Y_1 since there is no Y_0
  - $E(Y_1 | 1, A) = \delta_0 + \delta_1 A$

- Structural Model:
  - $Y_1 = \delta_0 + \delta_1 A + V$
  - $Y_t = \lambda_t + \theta Y_{t-1} + A + U_t$

Estimating θ

- Substitute for Y_1 in equation for Y_2
  - $Y_2 = \lambda_2 + \theta(\delta_0 + \delta_1 A + V) + A + U_2$
- Recursively substitute for t+1
- θ can be expressed as a function of Y_t and Y_s covariances

Reminders

- Not intended to be an exhaustive list
- Reference lecture notes and book for formulas or formal definitions
- Practice old homework and exam problems