

Economics Honors Exam 2007 Solutions Question 7

a. (5points) The profits of the imitator are:

$$px - L_x w = p \frac{1}{\chi} x - L_x w$$

the profits are zero iff

$$p = \chi w$$

b. (3 points) In order to make sure that the innovator is the only supplier of the intermediate good, the innovator should charge a price such that the profits of the imitator are zero. From a, this price is $p = \chi w$.

c. (5 points) If the innovator is successful, his profits are:

$$(p - w)x = (\chi w - w)L_x = w(\chi - 1)L_x$$

d. (2 points) The worker will make decision by comparing the wage he earns as a production worker with the expected profit he earns as an innovator. In other words, he compares w with $\lambda w(\chi - 1)L_x$.

e. (5 points) In equilibrium, as workers can freely decide their employment, their earnings must be the same in either sector (production or R&D), hence

$$\begin{aligned} w &= \lambda w(\chi - 1)L_x \\ \rightarrow L_x &= \frac{1}{\lambda(\chi - 1)} \\ \rightarrow L_A &= L - L_x = L - \frac{1}{\lambda(\chi - 1)} \end{aligned}$$

f. (7 points) The growth rate of technology in this economy is

$$\begin{aligned} g_A &= \frac{\dot{A}}{A} = \frac{A_{t+1} - A_t}{A_t} = \frac{[(1 - \lambda)^{L_A} A_t + (1 - (1 - \lambda)^{L_A}) \gamma A_t] - A_t}{A_t} \\ &= (\gamma - 1)(1 - (1 - \lambda)^{L_A}) = (\gamma - 1)(1 - (1 - \lambda)^{(L - \frac{1}{\lambda(\chi - 1)})}) \end{aligned}$$

The growth rate of GDP is

$$\begin{aligned} g_Y &= \frac{\dot{Y}}{Y} = \frac{A_{t+1} x^\alpha - A_t x^\alpha}{A_t x^\alpha} = g_A \\ &= (\gamma - 1)(1 - (1 - \lambda)^{(L - \frac{1}{\lambda(\chi - 1)})}) \end{aligned}$$

g. (3 points) If the imitator could produce the intermediate good at the same cost as the innovator, then the innovator cannot make profits from his innovation. Hence the innovator has no incentive to innovate, the growth rate in this economy will be 0.

Economics Honors Exam 2007 Solutions Question 8

a. (7 points) In this overlapping generation model, at date $t + 1$ total payroll tax on working people will be divided equally among retired population, therefore:

$$\tau y_{t+1}(N_{t+1} - N_t) = N_t b_{t+1}$$

or

$$\tau y_{t+1} n N_t = N_t b_{t+1} \rightarrow b_{t+1} = \tau(1+g)y_t n \quad (1)$$

b. (10 points) From (1), we have:

$$\begin{aligned} c_t &= (1-\tau)y_t - s \\ c_{t+1} &= (1+r)s + \tau(1+g)y_t n \end{aligned} \quad (2)$$

Plugging (2) in the objective function, the maximizing problem faced by a person born at date t is:

$$Max_s \ln((1-\tau)y_t - s) + \beta \ln((1+r)s + \tau(1+g)y_t n)$$

FOC:

$$\frac{-1}{(1-\tau)y_t - s} + \beta \frac{(1+r)}{(1+r)s + \tau(1+g)y_t n} = 0$$

\Rightarrow

$$-((1+r)s + \tau(1+g)y_t n) + \beta(1+r)((1-\tau)y_t - s) = 0$$

\Leftrightarrow

$$\beta(1+r)(1-\tau)y_t - \tau(1+g)y_t n = s(1+r)(1+\beta)$$

\Leftrightarrow

$$s = \frac{\beta(1+r)(1-\tau)y_t - \tau(1+g)y_t n}{(1+r)(1+\beta)} = \frac{y_t [\beta(1+r)(1-\tau) - \tau(1+g)n]}{(1+r)(1+\beta)} \quad (3)$$

→

$$\begin{aligned}
c_t &= (1 - \tau)y_t - s & (4) \\
&= (1 - \tau)y_t - \frac{y_t [\beta(1 + r)(1 - \tau) - \tau(1 + g)n]}{(1 + r)(1 + \beta)} \\
&= y_t \left[(1 - \tau) - \frac{[\beta(1 + r)(1 - \tau) - \tau(1 + g)n]}{(1 + r)(1 + \beta)} \right] \\
&= y_t \left[\frac{[(1 + r)(1 - \tau) - \tau(1 + g)n]}{(1 + r)(1 + \beta)} \right] \\
&= y_t \left[\frac{1}{(1 + \beta)} - \frac{\tau [(1 + r) - (1 + g)n]}{(1 + r)(1 + \beta)} \right]
\end{aligned}$$

and

$$\begin{aligned}
c_{t+1} &= (1 + r)s + \tau(1 + g)y_t n & (5) \\
&= (1 + r) \frac{y_t [\beta(1 + r)(1 - \tau) - \tau(1 + g)n]}{(1 + r)(1 + \beta)} + \tau(1 + g)y_t n \\
&= y_t \left[\frac{[\beta(1 + r)(1 - \tau) - \tau(1 + g)n]}{(1 + \beta)} + \tau(1 + g)n \right] \\
&= y_t \left[\frac{\beta(1 + r)}{(1 + \beta)} - \frac{\tau [\beta(1 + r) + (1 + g)n]}{(1 + \beta)} + \tau(1 + g)n \right] \\
&= y_t \left[\frac{\beta(1 + r)}{(1 + \beta)} - \frac{\tau [\beta(1 + r) + (1 + g)n]}{(1 + \beta)} + \tau(1 + g)n \right] \\
&= y_t \left[\frac{\beta(1 + r)}{(1 + \beta)} - \frac{\tau \beta [(1 + r) - (1 + g)n]}{(1 + \beta)} \right]
\end{aligned}$$

c. (3 points) From (3), it is easy to see that s is decreasing in τ . So if the government raises payroll tax to increase, savings will decrease.

d. (10 points) If there is no tax at all, then

$$\begin{aligned}
c_t &= y_t \left[\frac{1}{(1 + \beta)} \right] & (6) \\
c_{t+1} &= y_t \left[\frac{\beta(1 + r)}{(1 + \beta)} \right]
\end{aligned}$$

Comparing (4), (5) with (6), people are strictly better off or have higher life-time utility without tax ($\tau = 0$ and $b_{t+1} = 0$) iff

$$[(1 + r) - (1 + g)n] > 0$$

or saving rates is higher than total GDP growth.

Economics Honors Exam 2007 Solutions Question 9

(a) (5 points) The causal effect measures the expected increase in earnings as a result of participating in the training program.

(b) (10 points) Specifications (i) and (ii) can be written as follows:

$$(i) \quad y = \alpha + \beta d + u$$

$$(ii) \quad y = \alpha + \beta d + \gamma r + u$$

where u is the error term.

Specification (ii) controls for pre-program decision earnings while specification (i) does not control for this. Controlling for prior earnings is important if prior earnings are correlated with the participation decision and if prior earnings are a determinant of subsequent earnings. If both conditions are met, failing to control for prior earnings results in an omitted variable bias problem. It is very likely that these conditions are met in the real world. Therefore specification (ii) is more sensible than specification (i).

However, using specification (ii) does not necessarily provide a consistent estimate of the causal effect. Regression (ii) would provide a consistent estimate of the causal effect if the following condition is met:

$$E[u|d, r] = E[u|r]$$

In other words, if after conditioning on r , d is not correlated with the error term u , then regression (ii) would provide a consistent estimate of the causal effect. The condition would not hold true if some internal validity threats are still problematic. For instance, there could be other sources of omitted variable bias, such as 1) whether a person attended other training programs before or after this one, 2) whether the person is a single parent, 3) whether the person works part or full time, 4) person's profession, etc, 5) person's age, 6) years of prior work experience, etc. Another type of internal validity threat that could play an important role is misspecified functional form. It could be the case that training has a different impact on earnings, depending on one's career path. One could argue, for instance, that it is likely that the benefit of training on income would be greater for a financial analyst than a car mechanic. Thus, we may need to use an interaction of program participation with career to measure the effect. Furthermore, there could be an error in variable bias problem with using a binary variable for participation to measure the program training effect. Even though no one dropped out of the program, it could be that some people did better in the program than others. Perhaps other variables, such as

progress reports or grades could also be used to measure program participation.

(c) (15 points) If some of the internal validity threats mentioned in part (b) are problematic, we could address them using an instrumental variable approach. A variable (or a group of variables) constitutes a good instrument if it meets the following conditions:

- 1) relevancy - variable is correlated with d
- 2) exogeneity - variable is not correlated with the error term

If s meets these 2 conditions, it could be used in a 2-step procedure to instrument for d . s is likely to be relevant, because people given \$500 per month to participate in the program are likely more likely to participate. To test whether the relevancy condition is met, run the following regression: $d = \alpha + \beta s + \gamma r + u$ and test whether the F-statistic on β is higher than 10 (this is a rule of thumb). If it is, then s is relevant. It is likely that s is exogenous, because the subsidy is assigned randomly.

The benefit of using s as an instrument is that it allows us to address internal validity threats so that $E[u|d, r] = E[u|r]$ should hold after using the instrument. The cost is that if d and s are not strongly correlated, the program effect could be underestimated.

To construct an estimator for d using an instrumental variable approach, run the following regression: $d = \alpha + \beta s + \gamma r + u$, get estimates $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\gamma}$ and using these estimates get the predicted value \hat{d} . Then use \hat{d} in place of d in regression specification (ii), running the regression: $y = \mu + \lambda \hat{d} + \rho r + \varepsilon$. λ measures the causal effect of the training program of subsequent earnings. In order to estimate $\hat{\lambda}$, adjust standard errors to take into account that \hat{d} is a predicted value.

Economics Honors Exam 2007 Solutions Question 10

(a) (20 points)

Solving for $\hat{\lambda}$

$$\lambda = E(Y_{it}|A_i) - A_i$$

$$\lambda = E(\lambda) = E(E(Y_{is}|A_i) - A_i) = E(Y_s) - 0$$

$$\widehat{\lambda} = \frac{1}{3N} \sum_{i=1}^N \sum_{s=1}^3 Y_{is}$$

To show that $\widehat{\lambda}$ is a consistent estimate of λ , we must show that $\widehat{\lambda}$ converges in probability to λ as sample size increases.

$$\widehat{\lambda} = \frac{1}{3N} \sum_{i=1}^N \sum_{s=1}^3 Y_{is} = \frac{1}{3N} \sum_{i=1}^N Y_{i1} + \frac{1}{3N} \sum_{i=1}^N Y_{i2} + \frac{1}{3N} \sum_{i=1}^N Y_{i3}$$

we know that $\frac{1}{N} \sum_{i=1}^N Y_{is} \xrightarrow{p} EY_{is}$ for $s=1,2,3$. Therefore, $\widehat{\lambda} \xrightarrow{p} \lambda$, which means that $\widehat{\lambda}$ is a consistent estimate of λ .

(Note that using just one period's income would also have yielded a consistent estimate $\widehat{\lambda}$.)

Solving for $\widehat{\sigma}_a^2$

We know that $Cov(U_{i1}, U_{i3}) = 0$ from the given conditions

Let us re-write U_{i1} and U_{i3} in terms of Y_{is} and A_i

$$\begin{aligned} Cov(U_{i1}, U_{i3}) &= E[(Y_{i1} - (\lambda + A_i)) \times (Y_{i3} - (\lambda + A_i))] \\ &\quad - E[Y_{i1} - (\lambda + A_i)] \times E[Y_{i3} - (\lambda + A_i)] \\ &= 0 \end{aligned}$$

$$E(Y_{is}A_i) = E[Y_{is}|A_i] \times E[A_i] = 0 \text{ since } E[A_i] = 0$$

$$E[Y_{is}] = E[E[Y_{is}|A_i]] = \lambda \text{ since } E[A_i] = 0$$

$$\begin{aligned} Cov(U_{i1}, U_{i3}) &= E[Y_{i1}Y_{i3}] - E[Y_{i1}\lambda] - E[Y_{i1}A_i] \\ &\quad - E[Y_{i3}\lambda] - E[Y_{i3}A_i] + E[(\lambda + A_i) \times (\lambda + A_i)] \\ &= 0 \end{aligned}$$

$$\text{since } E[A_i] = 0, \sigma_A^2 = E[A_i^2], \text{ hence } E[Y_{i1}Y_{i3}] - \lambda^2 + E[A_i^2] = 0$$

$$\text{therefore } \sigma_A^2 = \lambda^2 - E[Y_{i1}Y_{i3}]$$

$$\widehat{\sigma}_A^2 = \widehat{\lambda}^2 - \frac{1}{N} \sum_{i=1}^N Y_{i1}Y_{i3}$$

$\frac{1}{N} \sum_{i=1}^N Y_{i1}Y_{i3} \rightarrow^p E[Y_{i1}Y_{i3}]$ and $\hat{\lambda}^2 \rightarrow^p \lambda^2$ so $\widehat{\sigma}_A^2$ is a consistent estimate of σ_A^2 .

Solving for $\widehat{\sigma}_U^2$

$$\sigma_U^2 = \text{Var}[Y_{is} - (\lambda + A_i)]$$

$$\sigma_U^2 = E[(Y_{is} - (\lambda + A_i))^2] - E[(Y_{is} - (\lambda + A_i))]^2 = E[Y_{is}^2] - 2E[(Y_{is}(\lambda + A_i))] + E[(\lambda + A_i)^2]$$

$$\sigma_U^2 = E[Y_{is}^2] - \lambda^2 + \sigma_A^2$$

$$\widehat{\sigma}_U^2 = \frac{1}{3N} \sum_{i=1}^N \sum_{s=1}^3 Y_{is}^2 - \hat{\lambda}^2 + \widehat{\sigma}_A^2 - \frac{1}{N} \sum_{i=1}^N Y_{i1}Y_{i3}$$

$$\widehat{\sigma}_U^2 = \frac{1}{3N} \sum_{i=1}^N \sum_{s=1}^3 Y_{is}^2 - \frac{1}{N} \sum_{i=1}^N Y_{i1}Y_{i3}$$

$\frac{1}{N} \sum_{i=1}^N Y_{i1}Y_{i3} \rightarrow^p EY_{i1}Y_{i3}$ and $\frac{1}{3N} \sum_{i=1}^N \sum_{s=1}^3 Y_{is}^2 \rightarrow^p E[Y_{is}^2]$ so $\widehat{\sigma}_U^2$ is a consistent estimate of σ_U^2 .

Solving for $\widehat{\gamma}$

$$\gamma = \text{Cov}(U_{is}, U_{is+1}) \text{ for } s = 1, 2$$

$$\gamma = E(U_{is}U_{is+1}) - E(U_{is})E(U_{is+1}) \text{ for } s = 1, 2$$

$$\gamma = E[(Y_{is} - (\lambda + A_i)) \times (Y_{is+1} - (\lambda + A_i))]$$

$$\gamma = E[Y_{is}Y_{is+1}] - \lambda^2 + \sigma_A^2$$

$$\widehat{\gamma} = \frac{1}{2N} \sum_{i=1}^N \sum_{s=1}^2 Y_{is}Y_{is+1} - \hat{\lambda}^2 + \widehat{\sigma}_A^2$$

$$\widehat{\gamma} = \frac{1}{2N} \sum_{i=1}^N \sum_{s=1}^2 Y_{is}Y_{is+1} - \frac{1}{N} \sum_{i=1}^N Y_{i1}Y_{i3}$$

$\frac{1}{2N} \sum_{i=1}^N \sum_{s=1}^2 Y_{is}Y_{is+1} \rightarrow^p E[Y_{is}Y_{is+1}]$ and $\frac{1}{N} \sum_{i=1}^N Y_{i1}Y_{i3} \rightarrow^p E[Y_{i1}Y_{i3}]$ so $\widehat{\gamma}$ is a consistent estimate of γ .

(b) (10 points)

We know that $\text{Cov}(Y_1, Y_4) = 0$ as $\text{Cov}(U_1, U_4) = 0$

Knowing Y_1 tells us nothing about A_i or Y_4 . Therefore, the best guess of Y_4 is the unconditional expectation.

$$E(Y_{N+1,4}) = E[E(Y_{i4}|A_i)] = \lambda$$

Therefore, $\widehat{Y}_{N+1,4} = \widehat{\lambda}$