a. Use equations I, II and III to write the accumulation equation for capital per effective unite of labor *m*

$$m = \frac{K}{AL}$$

$$\log - differenciating,$$

$$\ln(m) = \ln(K) - \ln(A) - \ln(L)$$

$$\frac{d\ln(m)}{dt} = \frac{d\ln(K)}{dt} - \frac{d\ln(A)}{dt} - \frac{d\ln(L)}{dt}$$

$$\frac{\dot{m}}{m} = \frac{\dot{K}}{K} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L}$$

$$with:$$

$$\frac{\dot{A}}{A} = g$$

$$\frac{\dot{L}}{L} = 0$$

$$\dot{K} = sY - \delta K$$

$$Y = K^{\alpha}(AL)^{1-\alpha}$$

$$substituting:$$

$$\frac{\dot{m}}{m} = \frac{sK^{\alpha}(AL)^{1-\alpha} - \delta K}{K} - g = s\left(\frac{K}{AL}\right)^{\alpha}\left(\frac{K}{AL}\right)^{-1} - \delta - g$$

$$\frac{\dot{m}}{m} = s\frac{m^{\alpha}}{m} - \delta - g$$

$$\dot{m} = sm^{\alpha} - (\delta + g)m$$

b. solve for the steady state level of *m*

$$\dot{m} = sm^{\alpha} - (\delta + g)m$$

in steady state:

$$\dot{m} = 0$$

$$sm^{\alpha} - (\delta + g)m = 0$$
$$sm^{\alpha-1} - (\delta + g) = 0$$
$$m^{\alpha-1} = \frac{(\delta + g)}{s}$$
$$m^{ss} = \left[\frac{(\delta + g)}{s}\right]^{\frac{1}{\alpha-1}}$$

c. What is the steady state growth of GDP per capita in this model?

In steady state:

$$\dot{m} = 0$$

$$m = \frac{K}{AL}$$
$$\log - \textit{differenciating},$$
$$\ln(m) = \ln(K) - \ln(A) - \ln(L)$$
$$\frac{d\ln(m)}{dt} = \frac{d\ln(K)}{dt} - \frac{d\ln(A)}{dt} - \frac{d\ln(L)}{dt}$$
$$\frac{\dot{m}}{m} = \frac{\dot{K}}{K} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L}$$
$$\textit{with}:$$
$$\frac{\dot{A}}{A} = g$$
$$\frac{\dot{L}}{L} = 0$$
$$\dot{m} = 0$$
$$\textit{substituting}:$$
$$\frac{\dot{K}}{K} = g$$
$$\textit{now},$$
$$\frac{\dot{Y}}{Y} = \frac{d\ln(Y)}{dt}$$
$$\ln(Y) = \alpha\ln(K) + (1-\alpha)\left[\ln(A) + \ln(L)\right]$$
$$\frac{d\ln(Y)}{dt} = \alpha\frac{\dot{K}}{K} + (1-\alpha)\left[\frac{\dot{A}}{A} + \frac{\dot{L}}{L}\right] = \alpha\frac{\dot{K}}{K} + (1-\alpha)\frac{\dot{A}}{A}$$
$$\textit{therefore},$$
$$\frac{\dot{Y}}{Y} = \alpha g + (1-\alpha)g = g$$

d. What happens to GDP per capita (y) when K falls (at time T)? Will the growth rate of GDP per capita be higher or lower?

$$Y = K^{\alpha}(AL)^{1-\alpha}$$

$$y = \frac{Y}{L} = \left(\frac{K}{L}\right)^{\alpha} A^{1-\alpha}$$

$$\ln y = \alpha(\ln K - \ln L) + (1-\alpha)\ln A$$

$$\frac{d\ln y}{d\ln K} = \varepsilon_{y,K} = \alpha$$

GDP per capita falls less than proportionally, when K falls.

$$y = \frac{Y}{L}$$

$$\log - \textit{differenciating}$$

$$\ln y = \ln Y - \ln L$$

$$\frac{d\ln y}{dt} = \frac{d\ln Y}{dt} - \frac{d\ln L}{dt}$$

$$\frac{\dot{y}}{y} = \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}$$

$$\textit{given},$$

$$\frac{\dot{L}}{L} = 0$$

$$\textit{then},$$

$$\frac{\dot{y}}{y} = \frac{\dot{Y}}{Y}$$

$$\textit{knowing},$$

$$\frac{\dot{y}}{y} = \frac{\dot{Y}}{Y} = \alpha\frac{\dot{K}}{K} + (1-\alpha)\frac{\dot{A}}{A}$$

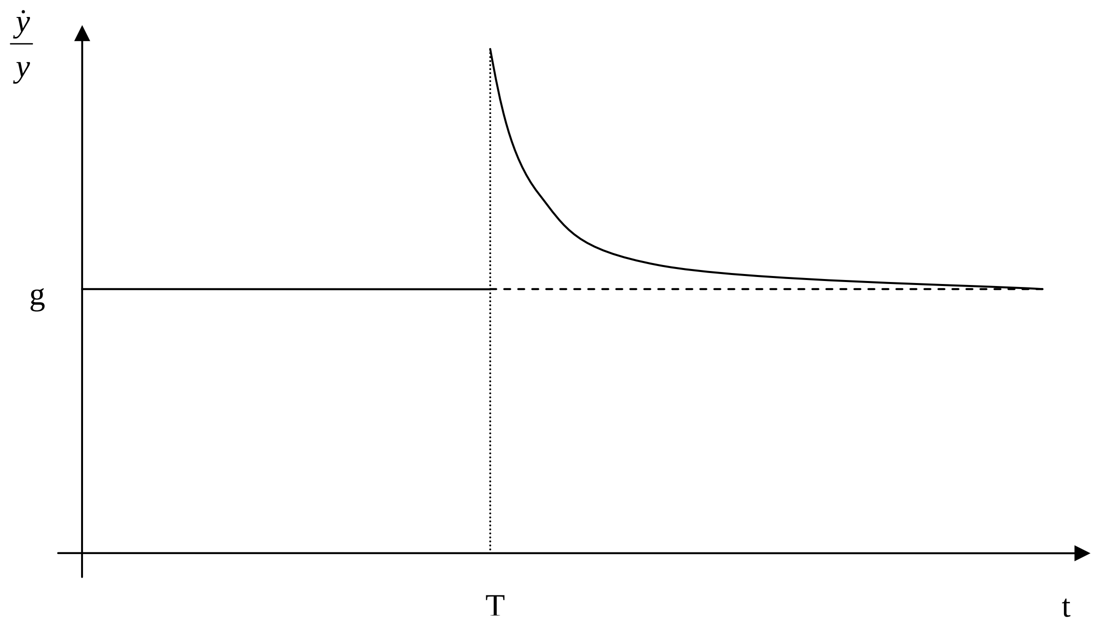
$$\frac{\dot{y}}{y} = \alpha\left(\frac{sY}{K} - \delta\right) + (1-\alpha)\frac{\dot{A}}{A}$$

$$\frac{\dot{y}}{y} = \alpha\left(sK^{\alpha-1}(AL)^{1-\alpha} - \delta\right) + (1-\alpha)\frac{\dot{A}}{A}$$

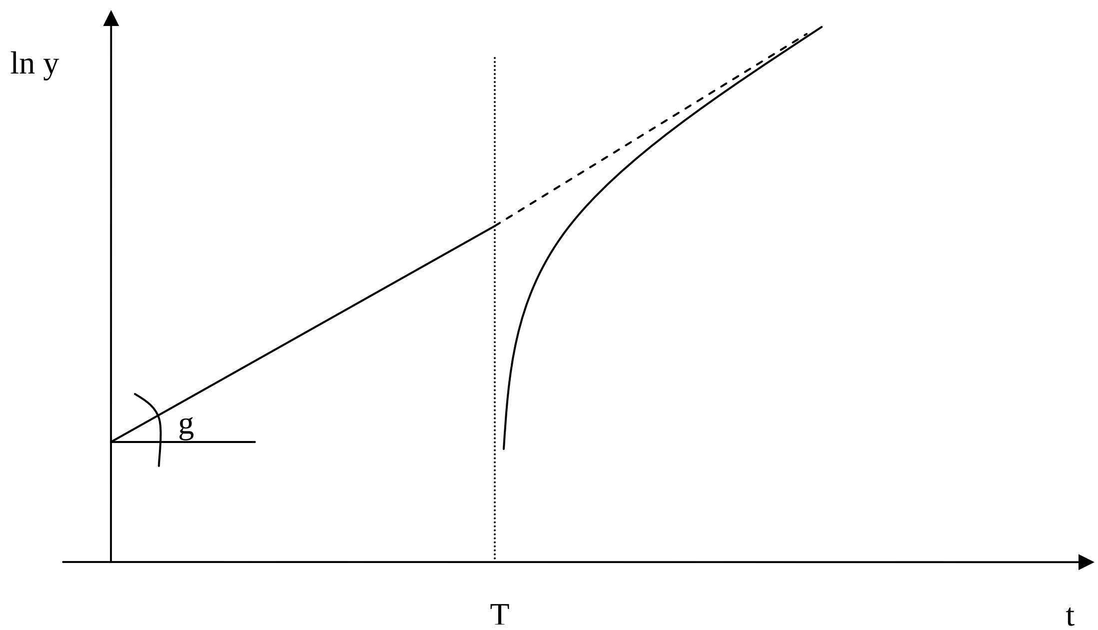
$$\frac{d\,\dot{y}/y}{dK} = (\alpha-1)\left[\alpha s(AL)^{1-\alpha}\right]K^{\alpha-2} \;\square\; 0$$

A decrease in K will increase the growth rate of GDP per capita.

e. Draw a graph with $\frac{\dot{y}}{y}$ on the vertical axis and time on the horizontal axis, and show the evolution of $\frac{\dot{y}}{y}$ before T and after.



f. Draw a graph with ln*y* on the vertical axis and time on the horizontal axis, and show the evolution of ln*y* before T and after.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