Macro Question 1

The (per capita) production function for the low technology is $y = Ak^a = k^{.5}$, where y and k are output and capital per worker respectively. The capital per worker changes according to the following equation $\dot{k} = sAk^a - (n+d)k = .1k^{.5} - .02k$. In the steady state $\dot{k} = 0$, and thus $5\sqrt{k} = k$ or $k = 25$. If the country starts with the initial capital stock per worker below 25, GDP per worker and capital per worker grow until they reach the steady state. If the country starts with k above 25 (but still uses the low technology), then GDP per worker declines until it achieves the steady state. The growth rate of GDP per capita is

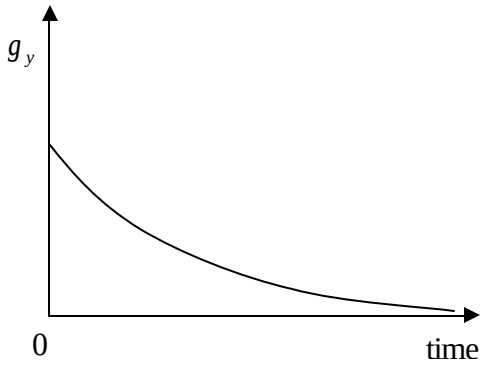
$$g_y = \frac{\dot{y}}{y} = \frac{.5k^{-.5}\dot{k}}{k^{.5}} = .5\frac{\dot{k}}{k} = .5\frac{.1k^{.5} - .02k}{k} = \frac{.05}{\sqrt{k}} - .01.$$

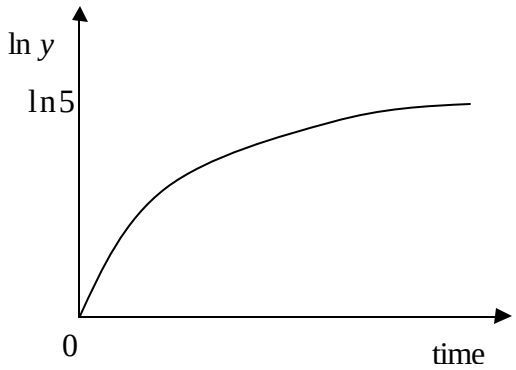
The (per capita) production function for the high technology is $y = Ak = k$. The dynamics of k is described by the following equation $\dot{k} = sAk - (n+d)k = .1k - .02k = .08k$. There is no steady state with the high technology both GDP per capita and capital per capita grow at the same constant rate

$$g_y = g_k = \frac{\dot{y}}{y} = \frac{\dot{k}}{k} = .08.$$

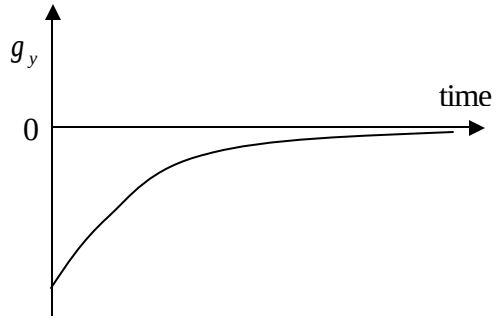
1. Now, we can draw the graphs.

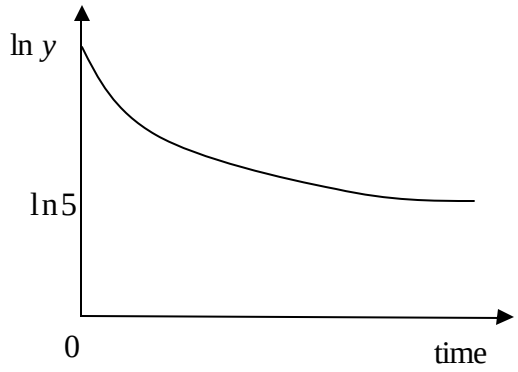
a) $k = 1$.



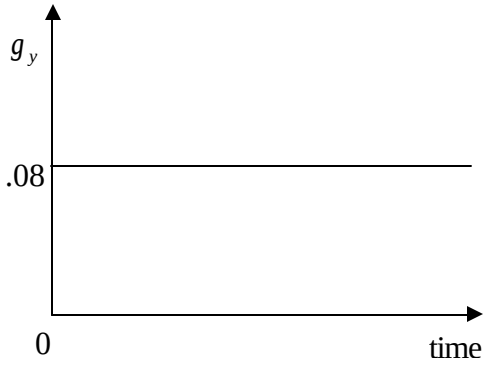


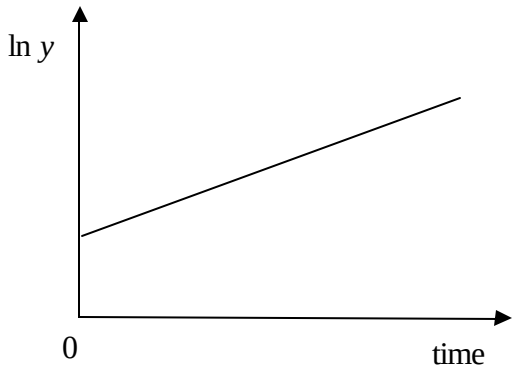
b) $k = 49$.





c) $k = 60$.





2. There will be no convergence in GDP per capita in the world. While all countries starting with a capital stock per capita below 50 will converge, the countries starting with k above 50 will be able to use the high technology and will grow without convergence. A plot of the growth rate of GDP per capita against initial GDP per capita will look as follows.